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Finite Difference Methods for Solving Linear and Nonlinear Ellipse Equations

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Abstract

The study aims to determine the numerical solution of partial differential equations of the ellipse type, both linear and non-linear. One of the most popular methods for solving partial differential equations is the finite differences method, which provides accurate approximations in comparison to the actual solution of the problem. The study provided clarification by utilizing the Laplace equation's numerical and analytical solution as one example of a partial differential equation of the linear ellipse kind. Regarding the study of solving partial differential equations of the type of nonlinear ellipse that is difficult to solve theoretically, we were satisfied with finding the formula only and programming it to obtain the required results. We then used the MATLAB system to find the solution to these equations. The results showed that the numerical solution is superior to the analytical solution because it can solve complex problems numerically, which is what the analytical solution cannot master accurately.

Introduction

Differential equations are the most effective means of describing the majority of geometric, mathematical (Redheffer & Port, 1992), and scientific problems, It is crucial to classify equations in order to investigate their solutions (Froese & Oberman, 2011). Ordinary differential equations and partial differential equations are two categories into which differential equations fall (Iserles, 2009). Recently, there has been a great deal of research focused on non-linear partial differential equations (LeVeque, 2007). Even with the growing advancement of mathematical ideas and techniques for resolving nonlinear partial differential equations (Bashier, 2020), numerical solutions As these solutions predominate in analytical solutions in this field, it is the most popular method for studying the properties of these equations (Zill et al., 1997).

In recent years, there has been an increase in the use of electronic computers to obtain numerical solutions to partial differential equations due to the extensive use of many highly effective methods (Shanthakumar, 1987). The commonly used partial differential equations are the second order partial differential equations whose general formula is (S H, 2012) .

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} + Fu + G = 0 \qquad (1)$$

where A, B, C, D, E, F and G are functions of(x) and (y) or constants

If $B^2 - 4AC < 0$ then Eq. (1) is called an ellipse equation

It is Examples Poisson's Equation In two dimensions (Adu, 2020):
$u_{xx} + u_{yy} = f(x, y)$

Laplace's Equation In two dimensions (Le Dret, 2018):

$u_{xx} + u_{yy} = 0$

Such that *A=1 ; B=0 ; C=1*

$\Delta = B^2 - 4AC$
$= 0 - 4(1)(1) = -4 < 0$

The function u(x,y) is the solution to this equation (Subramanian & White, 2004). It satisfies the solution of a point in the region R that is enclosed by a closed curve C as well as boundary conditions that are established on the closed curve C (Simmons & Krantz, 2007). The finite difference method is one of the most widely used techniques for solving partial differential equations. which, when using finite differences methods to solve problems, first divides the time needed to find the solution to (n-1) of the equal dimensional divisions (Ray, 2018). Next, we convert all of the derivatives that appear in the differential equation, as well as the derivatives that appear in the limit and prime value (Mathews & Fink Kurtis, 1999), into appropriate approximate differences. This method generally yields good approximations when compared to the real solution to the problem, then obtains a set of linear equations, the number of which equals the number of unknowns, and solves these equations concurrently using the proper algebraic techniques. We obtain the necessary numerical solution for solving systems of linear equations. Tyler's expansion is the most effective method for transforming the derivatives that arise in differential equations into suitable approximations (Le Dret, 2018), The expansion of Tyler will be continuous if the function (y) and its derivatives are functions of (x) :

$$y(x+h) = y(x) + hy'(x) + \frac{h^2}{2!}y''(x) + \cdots \qquad (2)$$

$$y(x-h) = y(x) - hy'(x) + \frac{h^2}{2!}y''(x) - \cdots \qquad (3)$$

Neglecting the higher ranks of (h) gives the Eq. (4) that gives the forward finite differences.

For the first derivative $y'(x)$ which is:

$$y'(x) = \frac{y(x+h) - y(x)}{h} \qquad (4)$$

Similarly, Eq. (5) gives the finite differences behind

$$y'(x) = \frac{y(x) - y(x-h)}{h} \qquad (5)$$

Subtracting Eq. (3) from Eq. (2) and neglecting the higher ranks of (h) gives us the central differences:

$$y'(x) = \frac{y(x+h) - y(x-h)}{h} \qquad (6)$$

Now by adding Eq.(3) to Eq.(2) we get:

$$y(x+h) + y(x-h) = 2y(x) + h^2 y''(x) + O(h^4)$$

where $O(h^4)$ represents the terms of the fourth order or more of (h) and neglecting the higher orders of (h) we get:

$$y''(x) = \frac{1}{2h}\big(y(x+h) - 2y(x) + y(x-h)\big) \qquad (7)$$

Since Eq. (7) represents the approximate central differences of the second derivative $y''(x)$, and to find the approximates of partial derivatives by the method of finite differences, we assume that (u) is a function of (x) and (y) and then divide the plane $(x$-$y)$ into a network of rectangles with sides $(\Delta x = h)$ and $(\Delta y = k)$ draws a set of lines so that:

$x = ih \quad ; i = 0,1,2,3, \ldots$

$y = jk \quad ; j = 0,1,2,3, \ldots$

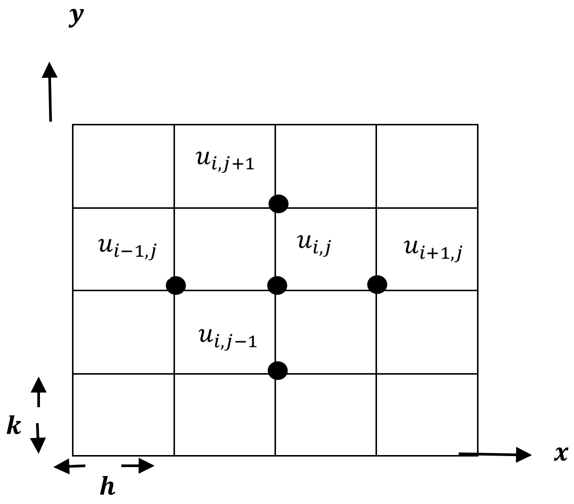


Fig. 1 *Grid Points (Jovanović & Süli, 2013)*

The points of intersection of these lines are known as grid points. The general method of solving partial differential equations by finite difference method is to obtain the solution at the grid points, and the grid points here may be caused by the intersection of rectangular lines in the case of $(h \neq k)$ or the intersection of square lines $(h = k)$. Therefore, the numerical approximation of the function's initial and secondary partial derivatives (u) is relative to (x, y) and using Tyler's expansion:

$$\frac{\partial u}{\partial x} = \frac{u_{i+1,j} - u_{i-1,j}}{2h} \qquad (8)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} \qquad (9)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} \qquad (10)$$

$$\frac{\partial u}{\partial y} = \frac{u_{i,j+1} - u_{i,j-1}}{2k} \qquad (11)$$

Therefore, partial differential equations can be converted into difference equations using the previous equations.

Numerical Solution of Linear Ellipse Equations

Examples include two-dimensional ellipse equations **Laplace equation:**

Laplace equation can be converted $u_{xx} + u_{yy} = 0$ and for a rectangular grid with dimensions (h, k) to the equation of differences (Ferziger & Perić, 2002) , where divide the rectangle $R = \{(x, y): 0 \leq x \leq a, 0 \leq y \leq b\}$ to a grid made up of sides-by-side $(n-1)(m-1)$ rectangles $\Delta x = h$, $\Delta y = k$ As seen in Fig. (2) :

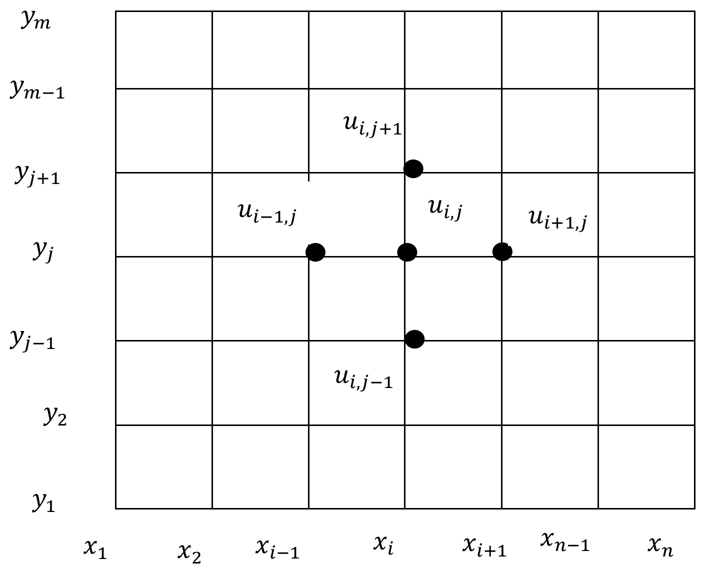


Fig. ***2*** *Shows the clip used in the Laplace difference equation*

We start calculating the first line when $y = y_1 = 0$ And the solution is defined by

$f(x_i) = u(x_i, y_1)$ The clasp space is uniform in each line so that:

$x_{i+1} = x_i + h$, $x_{i-1} = x_i - h$

It is uniform in each column so that:

$y_{j+1} = y_j + k$, $y_{j-1} = y_j - k$

We will convert the Laplace equation to a difference equation using the two Eq. (9) , Eq. (10) and substitute it in the Laplace equation as follows:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad (12)$$

Assuming that the grid is square, $(h = k)$ Therefore Eq. (12) can be written as follows:

$$u_{i,j} = \frac{1}{4}\left(u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1}\right) \qquad (13)$$

We can see from Eq. (13) that the value of $(u_{i,j})$ is the rate of the adjacent four points, as well as it is called: The standard five-point equation, It can be written as follows:

$$u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1} - 4u_{i,j} = 0 \qquad (14)$$

Sometimes this equation is also known as the Liebman's equation we can see the specific differences of this equation in the Fig. (1).

Thus, Eq. (13) or Eq. (14) can be developed for each point of the grid, given the boundary conditions, so we get equations as much as the number of unknowns (points) and then these equations are solved by one of the methods such as the Jacobi method or the Kaos Seidel method.

The equation similar to Eq. (13), which is sometimes also used, is:

$$u_{i,j} = \frac{1}{4}\left(u_{i-1,j-1} + u_{i+1,j+1} + u_{i+1,j-1} + u_{i-1,j+1}\right) \qquad (15)$$

This equation is known as the diagonal five-point equation. It can be noted here that although Eq.(15) is less accurate than e Eq.(13), it serves to give a good approximation to obtain initial values for the repetition process. we note the specific differences of Eq. (15) shown as in Fig. (3):

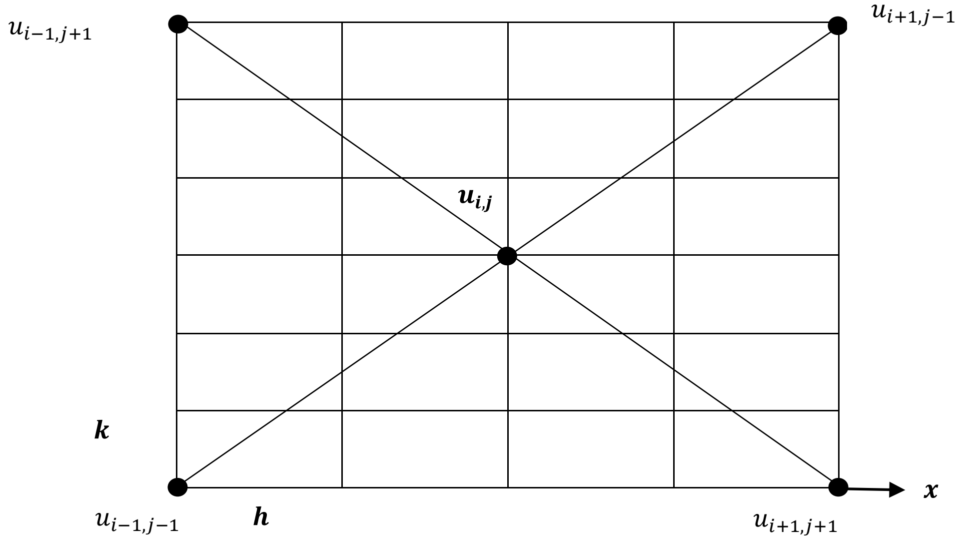


Fig. 3 *specific differences of the diagonal five-point equation*

Fig. 3 *Specific differences of the diagonal five-point equation*

To start the calculation steps, it is important to start with appropriate initial values, and this is by taking the largest length of the grid, and using the equation of the five diagonal points, Eq. (15), we obtain the initial value of the center point. For example To determine the values of (u_i) at the points of the internal network, then the values of (b_i) on the boundary are known as shown in Fig.(4).

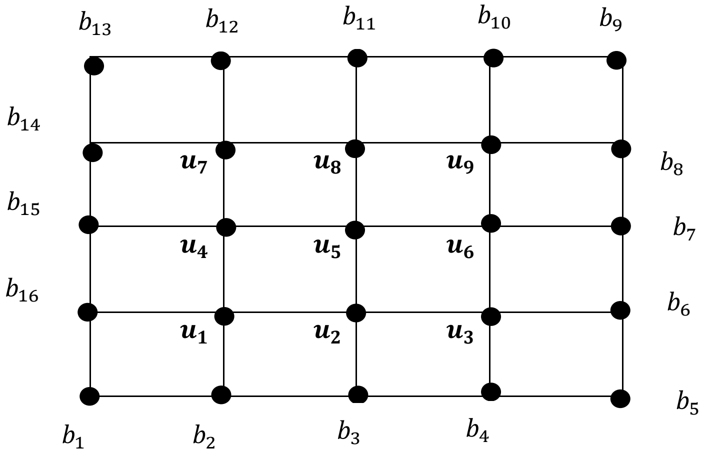


Fig. 4 *The points of the internal network*

When applying the five-point equation diagonal Eq.(15), we get the initial value of (u_5) at the center:

$$u_5 = \frac{1}{4}(b_1 + b_5 + b_9 + b_{13})$$

By knowing (u_5), the approximate values of (u_9, u_7, u_1, u_3) are calculated similarly using the equation of the five diagonal points Eq. (15) and the remaining values (u_8, u_6, u_4, u_2) we get them using the standard five-point Eq. (13) and then the process of applying either the Jacobi method or the Kaos Seidel method to obtain the final solution with the required accuracy and that the system $A_P = B$ lies solved by direct methods or by repetitive methods ، The method of deleting Kaos will be used to solve this system.

It should be noted that the boundary conditions either the information of any values known on the boundary such as (b_i) in the previous Fig. (4) or is partial derivatives which conditions are partial derivatives with respect to $u(x,y)$ at the terms. First of all, we equal the derivative with zero.

$$\frac{\partial}{\partial x} = u(x,y) = 0$$

We assume $x = x_i$ and the right-hand side $x = a$ for the rectangular

$$R = \{(x,y): 0 \le x \le a\, , 0 \le y \le b\}$$

Where the restrictions on boundaries are :

$$\frac{\partial}{\partial x} = u\left(x_i\, , y_j\right) = u_x\left(x_i\, , y_j\right) = 0$$

$$u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j} = 0 \qquad (16)$$

When $(u_{i+1,j})$ is a specific value and the reason is that it lies outside the R region, you can find its value using the formula of partial differential equations using the Eq. (8) we get:

$$\frac{u_{i+1,j} - u_{i-1,j}}{2h} \approx u_x\left(x_i\, , y_j\right) = 0$$

The value of the rounding is:

$$u_{i+1,j} \approx u_{i-1,j}$$

Substituting into Eq. (16) gives us

$$2u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j} = 0$$

Then we find the approximate values of $(u_{i,j})$ as mentioned earlier.

Numerical Solution of a Nonlinear Ellipse Equation

The nonlinear equation of an ellipse will be studied. which is in the form

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = u^2 \qquad (17)$$

Where we notice from Eq.(17) that the right side is a quadratic function and to solve such an equation with the availability of boundary conditions.

We will rely on substituting the numerical approximation of the second derivative of the function (u) relative to (x) and (y) using Tyler's expansion , we substitute Eq.(9) and the Eq.(10) into Eq. (17) to form :

$$\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = (u_{i,j})^2 \qquad (18)$$

In Eq.(18), we find the conversion of the nonlinear equation into a difference equation.

When assuming that the grid is square, *(h =k)* and therefore Eq.(18) can be written as follows:

$$u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j} = h^2\left(u_{i,j}\right)^2 \qquad (19)$$

Eq.(19) can be written as follows:

$$u_{i,j} = \frac{1}{4}\left(u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - h^2\left(u_{i,j}\right)^2\right) \qquad (20)$$

We note that Eq.(20) is difficult to solve theoretically , So we will suffice to find the formula only and program it to obtain the required results.

An example showing the numerical solution of the nonlinear equation:

Find the approximate numerical solution of the nonlinear equation:

$$\varepsilon^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = u^2 \qquad (21)$$

Where :

$$\varepsilon = \frac{h}{k}$$

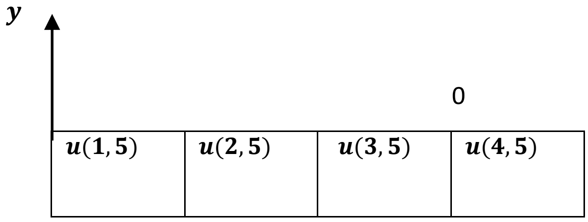
For the rectangular $R = \{(x, y) : 0 \le x \le 1 \;\; ; \;\; 0 \le y \le 1\}$

in relation to the function $u(x, y)$ at the boundary conditions:

$$\frac{\partial u}{\partial y}(x, 0) = 0 \quad \& \quad u(x, 1) = 0 \quad ; \quad 0 \le x \le 1$$

$$u(0, y) = 1 \quad \& \quad \frac{\partial u}{\partial y}(1, y) = 0 \quad ; \quad 0 \le y \le 1$$

Solution :



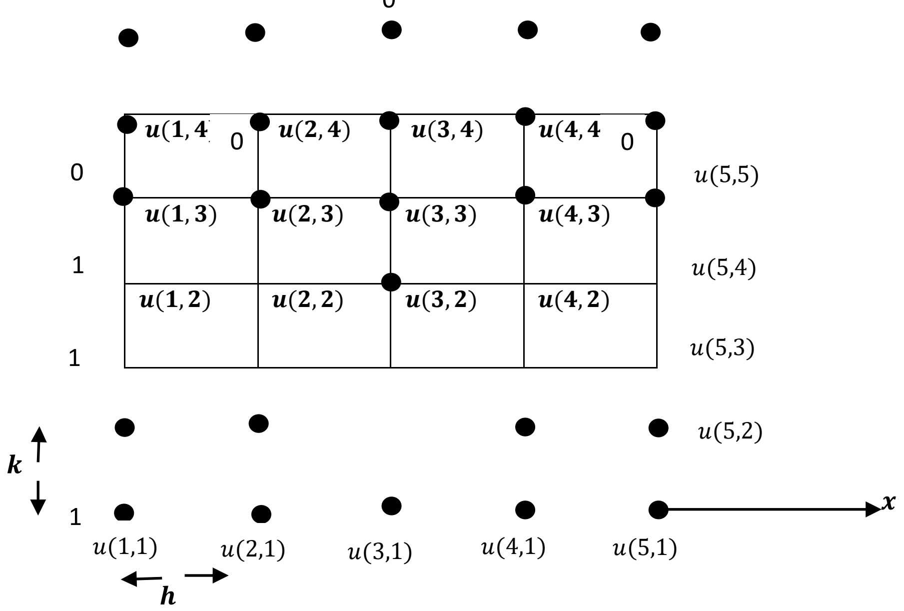


Fig. 5 *The function* $u(x, y)$ *at the boundary conditions*

We notice in the boundary conditions that there are boundary conditions that are partial derivatives where:

$$\frac{\partial u}{\partial y}(x, 0) = 0$$

$$\Longrightarrow \quad \frac{u_{i,j-1} + u_{i,j+1}}{2k} = 0 \quad \Longrightarrow \quad u_{i,j-1} = u_{i\,,j+1} \qquad (22)$$

$$\frac{\partial u}{\partial y}(1, y) = 0$$

$$\Longrightarrow \quad \frac{u_{i-1,j} + u_{i+1,j}}{2k} = 0 \quad \Longrightarrow \quad u_{i-1,j} = u_{i+1,j} \qquad (23)$$

We also find:

$\because$ h = 0.25 ; k = 0.25

$$\therefore \ \varepsilon = \frac{h}{k} = 1$$

Substituting in Eq.(21) the value of (ε) and converting the nonlinear equation into a difference equation obtains:

$$\Rightarrow \ (1)^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = u^2$$

$$\Rightarrow \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = (u_{i,j})^2$$

$$\Rightarrow u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1} - 4u_{i,j} = \frac{1}{16}(u_{i,j})^2$$

$$\Rightarrow u_{i,j} = \frac{1}{4}\left(u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1} - \frac{1}{16}(u_{i,j})^2\right) \qquad (24)$$

Substituting Eq.(22) and Eq.(23) into Eq.(24) results:

$$u_{i,j} = \frac{1}{4}\left(2u_{i-1,j} + 2u_{i,j-1} - \frac{1}{16}(u_{i,j})^2\right) \qquad (25)$$

Eq.(25) is the final formula.

Practical example:

Use finite difference methods to solve the following linear ellipse equation:

$$\varepsilon^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad ; \qquad \varepsilon = \frac{k}{h}$$

Under the following border conditions:

$u(x,0) = 0$; $u(x,1) = \sin h(\varepsilon\pi)\sin(\varepsilon\pi)$; $0 \le x \le 1$

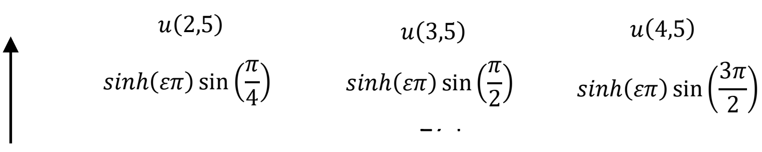
$u(0,y) = 0$; $u(1,y) = 0$; $0 \le y \le 1$

Solve the above equation in two ways:

(a) Numerical method

(b) Analytical method

Solution (a):



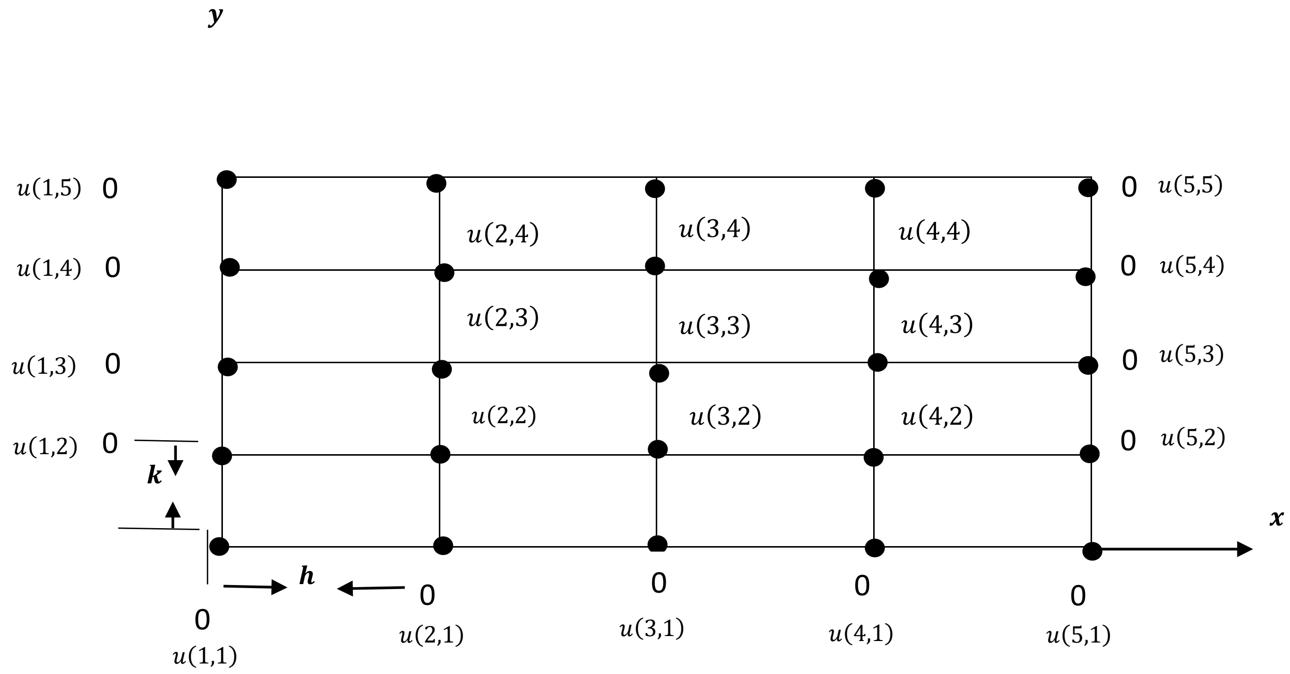


Fig.6 *The function* $u(x,y)$ *at the boundary conditions*

To find the solution we consider $h = 0.25$, $k = 0.25$ where we get : $\varepsilon = \frac{k}{h} = 1$

This leads to:

$$(1)^2 u_{xx} + u_{yy} = 0$$

Using Eq.(15) the equation of the five diagonal points:

$$u_{i,j} = \frac{1}{4}\left(u_{i-1,j-1} + u_{i+1,j-1} + u_{i+1,j+1} + u_{i-1,j+1}\right)$$

We count : $u(4,4)$, $u(2,4)$, $u(4,2)$, $u(2,2)$, $u(3,3)$

We get : $u(4,4)$=2.8872 , $u(2,4)$=2.887 , $u(4,2)$=0 , $u(2,2)$=0 , $u(3,3)$=0

Using Eq.(13) the equation of the standard five points:

$$u_{i,j} = \frac{1}{4}\left(u_{i-1,j} + u_{i+1,j} + u_{i,j+1} + u_{i,j-1}\right)$$

We count : $u(2,3)$, $u(3,4)$, $u(4,3)$, $u(3,2)$

We get : $u(2,3)$=0.7218 , $u(3,4)$=4.3308 , $u(4,3)$=0.7218 , $u(3,2)$=0

Applying the equation of the five standard points to each of the internal points of the clamp, this generates a set of (9×9) linear equations:

$$u_{2,2} = \frac{1}{4}\left(0 + u_{3,2} + u_{2,3} + 0\right) \qquad (26)$$

$$u_{3,2} = \frac{1}{4}\left(0 + u_{4,2} + 0 + u_{3,3}\right) \qquad (27)$$

$$u_{4,2} = \frac{1}{4}\left(u_{3,2} + 0 + u_{4,3} + 0\right) \qquad (28)$$

$$u_{2,3} = \frac{1}{4}\left(0 + u_{3,3} + u_{2,4} + u_{2,2}\right) \qquad (29)$$

$$u_{3,3} = \frac{1}{4}\left(u_{2,3} + u_{4,3} + u_{3,4} + u_{3,2}\right) \qquad (30)$$

$$u_{4,3} = \frac{1}{4}\left(u_{3,3} + 0 + u_{4,4} + u_{4,2}\right) \qquad (31)$$

$$u_{2,4} = \frac{1}{4}\left(u_{3,4} + 8.1662 + u_{2,3}\right) \qquad (32)$$

$$u_{3,4} = \frac{1}{4}\left(u_{2,4} + u_{4,4} + 11.5487 + u_{3,3}\right) \qquad (33)$$

$$u_{4,4} = \frac{1}{4}\left(u_{3,4} + 8.1662 + u_{4,3}\right) \qquad (34)$$

After the following coding :

$u_{2,2}=u_1^n=0$

$u_{3,2}=u_2^n=0$

$u_{4,2}=u_3^n=0$

$u_{2,3}=u_4^n=0.7218$

$u_{3,3}=u_5^n=0$

$u_{4,3}=u_6^n=0.7218$

$u_{2,4}=u_7^n=2.8872$

$u_{3,4}=u_8^n=4.3308$

$u_{4,4}=u_9^n=2.8872$

Applying the Kaos Seidel method :

$$u_1^{(n+1)} = \frac{u_2^n + u_4^n}{4} \qquad (35)$$

$$u_2^{(n+1)} = \frac{u_1^{(n+1)} + u_3^n + u_5^n}{4} \qquad (36)$$

$$u_3^{(n+1)} = \frac{u_2^{(n+1)} + u_6^n}{4} \qquad (37)$$

$$u_4^{(n+1)} = \frac{u_5^{(n)} + u_7^n + u_1^{(n+1)}}{4} \qquad (38)$$

$$u_5^{(n+1)} = \frac{u_4^{(n+1)} + u_6^n + u_8^n + u_5^{(n+1)}}{4} \qquad (39)$$

$$u_6^{(n+1)} = \frac{u_5^{(n+1)} + u_9^n + u_3^{(n+1)}}{4} \qquad (40)$$

$$u_7^{(n+1)} = \frac{u_8^n + u_4^{(n+1)} + 8.1662}{4} \qquad (41)$$

$$u_8^{(n+1)} = \frac{u_7^{(n+1)} + u_9^n + u_5^{(n+1)} + 115487}{4} \qquad (42)$$

$$u_9^{(n+1)} = \frac{u_8^{(n+1)} + u_6^{(n+1)} + 8.1662}{4} \qquad (43)$$

Where is (*n = 1 , 2 , 3 , …*) to evaluate and fix the initial values where the arithmetic iteration can continue to obtain the final results.

Solution (b):

In this way we substitute the value of both x , y in :

$u = \sin h(\varepsilon\pi y)\sin(\pi x) \quad ; \ 0 \le x \le 1 \quad ; \quad 0 \le \mathrm{y} \le 1$

This is by dividing the period [0,1] where $h = 0.25$; $\varepsilon = 1$

As in Table (2) Results of the analytical method (b) .

Tables :

Table 1: Results of the numerical method (a)

$u(x_i, y_j)$	x_1	x_2	x_3	x_4	x_5
y_5	0	8.1662	11.5487	8.1662	0
y_4	0	3.8320	5.4193	3.8320	0
y_3	0	1.7425	2.4643	1.7425	0
y_2	0	0.6739	0.9530	0.6739	0
y_1	0	0	0	0	0

Table 2: Results of the analytical method (b)

$u(x_i, y_j)$	x_1	x_2	x_3	x_4	x_5
y_5	0	8.1661	11.3487	8.1661	0
y_4	0	3.6967	5.22797	3.6967	0
y_3	0	1.6272	2.3013	1.6272	0
y_2	0	0.6143	0.8687	0.6143	0
y_1	0	0	0	0	0

Table 3 :Comparison between numerical and analytical method when j=3

$Analytic\ M.$	FDM	$error$
0	0	0

1.6272	1.7425	0.1153
2.3013	2.4643	0.163
1.6272	1.7426	0.1154
0	0	0

Table 4 :Numerical results of nonlinear Eq.(21)

$u(x_{i\,,}\,y_j)$	x_1	x_2	x_3	x_4	x_5
y_5	0	0	0	0	0
y_4	1	0.4285	0.1875	0.0714	0
y_3	1	0.5267	0.2499	0.0982	0
y_2	1	0.4284	0.1874	0.0714	0
y_1	1	0	0	0	0

The shapes :

1)

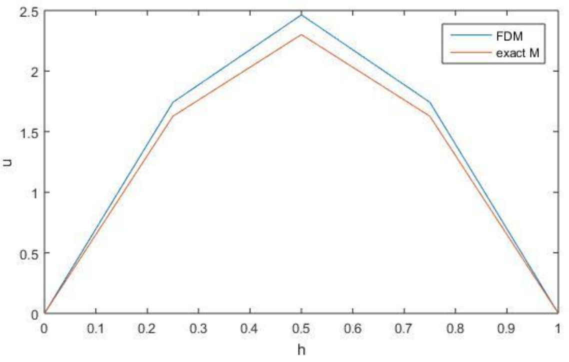
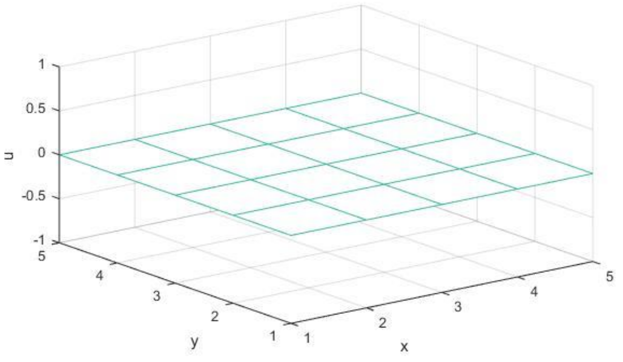


Fig. 7 *Comparison between numerical and analytical method when j=3*



2)

Fig. 8 *The numerical solution using the numerical method of the nonlinear equation*

Conclusion

Finite difference methods are a numerical method used to solve partial differential equations such as the linear and nonlinear ellipse equation. These methods are based on approximating partial derivatives using finite differences between grid points distributed over the solution field ، The linear ellipse equation using finite difference methods was converted from a partial differential equation into a system of linear algebraic equations. The accuracy of the solution depends on the size of the grid and the number of points used, and that the numerical solution is close to the real solution ، The non-linear ellipse equation adds a level of complexity to the solution. Therefore, the initial values must be carefully chosen to ensure convergence to the correct solution. The accuracy of the solution depends on the size of the network and the number of points, but non-linear solutions may be more sensitive to these factors.

References:

1. Adu, P. (2020). *Modified Iterative Method for Computing the Approximate Solutions of Nonlinear Equations* University of Cape Coast].

2. Bashier, E. B. (2020). *Practical Numerical and Scientific Computing with MATLAB® and Python*. CRC Press.

3. Ferziger, J. H., & Perić, M. (2002). *Computational methods for fluid dynamics*. Springer.

4. Froese, B. D., & Oberman, A. M. (2011). Convergent finite difference solvers for viscosity solutions of the elliptic Monge–Ampère equation in dimensions two and higher. *SIAM Journal on Numerical Analysis*, *49*(4), 1692-1714.

5. Iserles, A. (2009). *A first course in the numerical analysis of differential equations*. Cambridge university press.

6. Jovanović, B. S., & Süli, E. (2013). *Analysis of finite difference schemes: for linear partial differential equations with generalized solutions* (Vol. 46). Springer Science & Business Media.

7. Le Dret, H. (2018). *Nonlinear elliptic partial differential equations*. Springer.

8. LeVeque, R. J. (2007). *Finite difference methods for ordinary and partial differential equations: steady-state and time-dependent problems*. SIAM.

9. Mathews, J., & Fink Kurtis, D. (1999). Numerical Methods Using Matlab, Prenctice-Hall. *Inc. New York*.

10. Ray, S. S. (2018). *Numerical analysis with algorithms and programming*. Chapman and Hall/CRC.

11. Redheffer, R. M., & Port, D. (1992). *Introduction to differential equations*. Jones & Bartlett Learning.

12. S H, L. (2012). *Numerical analysis of partial differential equations*. John Wiley & Sons.

13. Shanthakumar, M. (1987). *Computer Based Numerical Analysis*. Khanna.

14. Simmons, G. F., & Krantz, S. G. (2007). *Differential equations: theory, technique, and practice*. McGraw-Hill New York.

15. Subramanian, V. R., & White, R. E. (2004). Semianalytical method of lines for solving elliptic partial differential equations. *Chemical engineering science*, *59*(4), 781-788.
16. Zill, D. G., Cullen, M. R., & Wright, W. S. (1997). *Differential equations with boundary-value problems*. Brooks/Cole Publishing Company.

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طرائق الفروقات المنتهية لحل معادلة القطع النّاقص الخطيّة وغير الخطيّة

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الخلاصة:

هدفت الدراسة إلى معرفة الحل العددي للمعادلات التفاضلية الجزئية من نوع القطع الناقص لكلا النوعين الخطي وغير الخطي ومن أكثر الطرق استخدامًا لحلّ المعادلات التفاضلية الجزئية طريقة الفروق المنتهية والتي تعطي تقديرات تقريبية جيدة مقارنة بالحلّ الحقيقي للمسألة. أوضحت الدراسة باستخدام الحل العددي والتحليلي لمعادلة لابلاس كأحد أنواع المعادلات التفاضلية الجزئية من نوع القطع الناقص الخطي. وتبيّن أنّ الحل العددي أفضل من الحل التحليلي بسبب قدرته على حل المسائل المعقدة عدديًا وهذا ما لا يمكن إتقانه بدقة في الحل التحليلي، أمّا الدراسة في حل المعادلات التفاضلية الجزئية من نوع القطع الناقص غير الخطي الذي يصعب حله نظريًا، فقد اكتفيت بإيجاد الصيغة فقط وبرمجتها للحصول على النتائج المطلوبة، واستخدم نظام ماتلاب لإيجاد حل هذه المعادلات.