IRAQI Academic Scientific Journals

Samarra Journal of Pure and Applied Science

www.sjpas.com

p ISSN: 2663-7405
e ISSN: 2789-6838

# Using the Developed Conjugate Gradient Algorithm and the Sand Cat Swarm Optimization (SCSO) Algorithm to Improve the Performance of the Whale Optimization Algorithm (WOA)

**Omar Dhahir Shalal [1,2*] and Ban Ahmed Mitras [1]**

1- Department of Mathematics, College of Computer Science and Mathematics, University of Mosul, Iraq
2- Nineveh Education Directorate, Ministry of Education, Nineveh, Iraq



https://doi.org/10.54153/sjpas.2024.v6i4.918

**Article Information**

Received: 28/04/2024
Revised: 31/05/2024
Accepted: 02/06/2024
Published:30/12/2024

**Keywords:**

*Metaheuristic algorithm, Conjugate Gradient Algorithm, Sand Cat Swarm Optimization Algorithm (SCSO), Whale Optimization Algorithm (WOA), Hybrid Algorithms.*

**Corresponding Author**

E-mail: omar.21csp24@student.uomosul.edu.iq
Mobile: +9647716895052

**Abstract**

In this paper, two distinct strategies were used to enhance problem-solving abilities. The first strategy involved developing a conjugate gradient algorithm such that a new parameter was extracted and proposed. The second strategy included Improving the Whale Algorithm (WOA) in two ways, the first using the community by taking advantage of the developed conjugate gradient algorithm that was extracted from the first strategy and obtaining the proposed algorithm (CG-WOA) that gives better results than the results from the original algorithm. The second method is to combine the sand cat swarm optimization algorithm (SCSO) with the whale optimization algorithm (WOA), and the proposed algorithm (SCSO-WOA) is obtained. The proposed algorithms (CG-WOA) and (SCSO-WOA) have many characteristics, including their ability to deal with complex optimization problems and their speed efficiency compared to the original algorithm. The diversity of exploration and exploitation operations in the proposed algorithm (SCSO-WOA) gives it the advantage of fast convergence, obtaining the global optimal solution, and avoiding falling into local solutions. The efficiency of the two algorithms (CG-WOA) and (SCSO-WOA) was tested on five standard test functions, so that better numerical results were obtained than the results of the original

## Introduction

Although many individual developmental algorithms inspired by nature through foraging and social behavioral ecology can obtain good solutions to many problems [1], [2], due to the rapid development in all fields, individual developmental algorithms may fail to solve some problems. The problems require the use of modern optimization or non-traditional optimization methods to solve complex optimization problems. This prompted many scientists to think about merging some algorithms with each other and obtaining a new algorithm that gives better results than the results of the two original algorithms.

Numerical methods for achieving optimization are designed to find a global minimum value at the real vector $x^* \in R^n$. If the $f : R^n \rightarrow R$ minimization of the objective function is expressed by the following formula:

$$\min f(x), x \in R^n \tag{1}$$

$$x_{k+1} = x_k + \lambda_k d_k , \quad k = 0,1,2,3, \dots \tag{2}$$

Where $\lambda_k > 0$ is the step size. To demonstrate global convergence, we apply the SWL search, which requires $\omega_{k-1}$ to satisfy certain conditions

$$f(x_{k+1} + \lambda_k d_k) \leq f(x_k) + \delta_1 \omega_k g_k^T d_k \tag{3}$$

$$|g(x_{k+1} + \lambda_k d_k)^T d_k| \leq \delta_2 |g_k^T d_k| \tag{4}$$

Where $0 < \delta_1 < \delta_2 < 1$ , are the parameters [3]. The conjugate gradient (CG) direction is defined by:

$$d_{k+1} = \begin{cases} -g_{k+1} , & k = 1 \\ -g_{k+1} + \beta_{k+1} d_{k,} & k \geq 2 \end{cases} \tag{5}$$

Optimization problems are divided into two parts: deterministic algorithms and stochastic algorithms [3]. Most of the traditional algorithms are deterministic algorithms. Stochastic algorithms have two types of algorithms: heuristic algorithms and metaheuristic algorithms. That is, metaheuristic algorithms are a class of random search approaches that display outstanding performance in solving multi-modal, non-differentiable problems [4]. [5],[6],[7]. Since metaheuristic algorithms are very common and have been widely used in solving optimization problems in the real world and in various fields such as computational computing [8], [9]. and scheduling [10]. and neural network [11]. and image segmentation [12]. and fuzzy control [13]. and civil engineering [14], [15]. etc. The work that we did was to hybridize the whale algorithm using two algorithms, Specifically the Formulated conjugate gradient algorithm that was derived in the first part of this paper, which is a Progress to the initial population of the whale algorithm and obtained the (CG-WOA) algorithm. The second hybridization was by the sand cat swarm optimization algorithm (SCSO), An algorithm that emulates the behavior of sand cats in nature, and is characterized by hearing low-frequency sounds and a distinctive ability to dig in search of prey [16], By merging it with the Whale Optimization Algorithm (WOA), which is an optimization algorithm inspired by nature that mimics the social behavior of whales. Therefore, the two proposed algorithms (CG-WOA) and (SCSO-WOA) can be used to solve a wide range of problems in various fields.

The general framework of this paper is as follows: Section 2 developed the conjugate gradient algorithm. Section 3 presents the sand cat swarm optimization (SCSO) algorithm. In Section 4, the Whale Optimization Algorithm (WOA) is presented. In Section 5 presents the proposed algorithms. Section 6 presents the analysis of the results. Either conclusion is presented in Section 7.

## Developed conjugate gradient algorithm

It is one of the mathematical methods used to solve problems of finding the minimum or maximum of functions. This method is usually used in problems searching for solutions to systems of linear equations or improving performance in numerical research problems. The conjugate gradient method is based on the use of conjugate directives to optimize the function being optimized quickly. Instead of using traditional regression directions, the conjugate gradient method uses directions that are interconnected with each other. You start moving towards the initial downhill direction. A conjugate direction of progress is then determined based on the previous step and the interconnected trends. This process is repeated until the solutions improve simultaneously in the directions of all regressions.

**Conjugate gradient algorithms [19]**

The conjugate gradient algorithm not only uses the conjugate direction, but instead calculates the direction $d_k$.

At each step of the algorithm, the trend is calculated $d_k$ as a linear combination of the previous trend and the current gradient, in a way that makes the trends conjugate two-by-two, which is why they are called conjugate gradient algorithms.

$$d_{k+1} = -g_{k+1} + \beta_k \, d_k \quad k \geq 0$$

As for the linear combination of the gradient direction in the case of three terms, presented by Zhang et. al. In 2006, as shown in the following formula: [19]

$$d_{k+1} = -g_{k+1} + \beta^{PR} \, d_k - \varphi \, y_k \quad k \geq 0$$

Where $\beta_k^{PR} = \frac{g_{k+1}^T \, y_k}{\|g_k\|^2}$ , $\varphi_k = \frac{g_{k+1}^T \, d_k}{\|g_k\|^2}$

**Derive the new variable $\boldsymbol{\beta_k^{New}}$**

Let:

$$d_{k+1}^{CG_2} = d_{k+1}^{CG_3}$$

$$-g_{k+1} + \beta_k^{New} \, d_k = -g_{k+1} + \beta^{PR} \, d_k - \varphi \, y_k$$

$$-g_{k+1} + \beta_k^{New} \, d_k = -g_{k+1} + \frac{g_{k+1}^T \, y_k}{\|g_k\|^2} \, d_k - \varphi \, y_k$$

$$-y_k^T \, g_{k+1} + \beta_k^{New} \, y_k^T \, d_k = -y_k^T \, g_{k+1} + \frac{g_{k+1}^T \, y_k}{\|g_k\|^2} \, y_k^T \, d_k - \varphi \, y_k^T \, y_k$$

$$\beta_k^{New} \, y_k^T \, d_k = \frac{g_{k+1}^T \, y_k}{\|g_k\|^2} \, y_k^T \, d_k - \varphi \, y_k^T \, y_k$$

$$\beta_k^{New} = \frac{g_{k+1}^T \, y_k}{\|g_k\|^2} \, \frac{y_k^T \, d_k}{y_k^T \, d_k} - \varphi \, \frac{\|y_k\|^2}{y_k^T \, d_k}$$

$$\beta_k^{New} = \frac{g_{k+1}^T \, y_k}{\|g_k\|^2} - \frac{g_{k+1}^T \, d_k}{\|g_k\|^2} \, \frac{\|y_k\|^2}{y_k^T \, d_k}$$

$$\beta_k^{New} = \frac{1}{\|g_k\|^2}\left[ g_{k+1}^T \, y_k - g_{k+1}^T \, d_k \, . \, \frac{\|y_k\|^2}{y_k^T \, d_k} \right]$$

$$\beta_k^{New} = \frac{1}{\|g_k\|^2}\left[ g_{k+1}^T \, y_k - \frac{g_{k+1}^T \, d_k}{y_k^T \, d_k} . \, \|y_k\|^2 \right] \quad (6)$$

**Steps of the proposed algorithm (modification conjugate gradient)**

**Step 1:** Choose a starting value $x_0$, set $d_0 = -g_0$, $k = 0$ and $\varepsilon > 0$.

**Step 2:** Calculate the step length $\lambda_k > 0$, satisfying Wolfe's condition.

**Step 3:** Calculate $x_{k+1} = x_k + \lambda_k d_k$. If $\|g_{k+1}\| < \varepsilon$, then stop.

**Step 4:** Calculate $\beta_k^{New}$ and generate the direction $d_{k+1} = -g_{k+1} + \beta_k^{New} d_k$.

**Step 5:** Set $k = k + 1$ and go to step 2.

**Convergence analysis of the new conjugate vector method**

In this section, we will show that the proposed algorithm that has been identified achieves the property of sufficient regression that satisfies the property of convergence

**Assumption (1):** If f is bounded on the set $s = \{x \in R^n: f(x) \leq f(x_0)\}$ and is differentiable with the gradient $\nabla f$ and there is a Lipchitz Constant $L > 0$, then
$\|\nabla f(x) - \nabla f(y)\| < L\|x - y\|$ for all $x, y \in s$.

**Theorem (1):** The search direction $d_k$ generated by the proposed algorithm of the developed CG achieves the property of sufficient steepness for each k, when the step size $\lambda_k$ satisfies Wolfe conditions.

**Proof:** Using the principle of mathematical induction

When $k=0$, $d_0 = -g_0 \Longrightarrow d_0^T g_0 = -\|g_0\| < 0$ then the theorem is true.

Now we assume that the theorem is true for all values of $k \geq 0$, i.e

$$g_k^T d_k < 0 \text{ ‘ } g_k^T d_k \leq -c\|g_k\|^2 \text{ ‘ } c > 0$$

We will now explain that the theorem is true when *k+1*.

By multiplying both sides of Eq. (7) below by $g_{k+1}^T$ we get:

$$d_{k+1} = -g_{k+1} + \beta_k^{New} d_k \tag{7}$$

$$g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \beta_k^{New} g_{k+1}^T d_k$$

$$g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \frac{1}{\|g_k\|^2}\left[ g_{k+1}^T\ y_k - \frac{g_{k+1}^T\ d_k}{y_k^T\ d_k}.\ \|y_k\|^2 \right] g_{k+1}^T d_k \tag{8}$$

From Wolff's second strong condition, as shown in Eq. (9) below:

$$\begin{gathered} g(x_k + \alpha_k d_k)^T\ d_k \leq -p\ g_k^T d_k \\ \Longrightarrow\ g_{k+1}^T\ d_k \leq -p\ g_k^T d_k \end{gathered} \tag{9}$$

It is known that $g_{k+1}^T\ y_k = g_{k+1}^T(g_{k+1} - g_k) = \|g_{k+1}\|^2 - g_{k+1}^T\ g_k$

Taking advantage of one direction of Powell's recovery condition, as shown in Eq. (10):

$$g_{k+1}^T\ y_k \leq \|g_{k+1}\|^2 + 0.2\ \|g_{k+1}\|^2 = 1.2\ \|g_{k+1}\|^2 \tag{10}$$

From Wolff's strong condition we get the following Eq. (11):

$$-(1-p) g_k^T d_k \leq y_k^T\ d_k \leq -(1+p) g_k^T d_k \tag{11}$$

Substituting equations (9), (10) and (11) into Eq. (8) results in:

$$g_{k+1}^T d_{k+1} \leq -\|g_{k+1}\|^2 + \frac{1}{\|g_k\|^2}\left[ 1.2\ \|g_{k+1}\|^2 - \frac{-p\ g_k^T d_k}{-(1-p) g_k^T d_k}.\ \|y_k\|^2 \right](-p\ g_k^T d_k)$$

$$g_{k+1}^T d_{k+1} \leq -\|g_{k+1}\|^2 + \frac{1}{\|g_k\|^2}\left[ 1.2\ \|g_{k+1}\|^2 - \frac{p\ g_k^T d_k}{(1-p) g_k^T d_k}.\ \|y_k\|^2 \right](cp\ \|g_k\|^2)$$

$$g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \left[ 1.2\ \|g_{k+1}\|^2 - \frac{p}{(1-p)}.\ \|y_k\|^2 \right](cp\ )$$

$$g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + 1.2cp\ \|g_{k+1}\|^2 - \frac{cp^2}{(1-p)}.\ \|y_k\|^2$$

$$g_{k+1}^T d_{k+1} = -(1 - 1.2cp)\|g_{k+1}\|^2 - \frac{cp^2}{(1-p)}.\ \|y_k\|^2$$

Since $c > 0$, $0.5 < p < 1$, therefore the part is $\frac{cp^2}{(1-p)}.\ \|y_k\|^2 > 0$ This leads to

$$g_{k+1}^T d_{k+1} \leq -(1 - 1.2cp)\|g_{k+1}\|^2$$

$$g_{k+1}^T d_{k+1} \leq -\Omega\|g_{k+1}\|^2 \qquad where\ \Omega = 1 - 1.2cp > 0. \text{ when } 0 < c < 1.$$

**Comprehensive convergence analysis of the proposed algorithm**

**Lemma (1):** Suppose that Assumption (1) is fulfilled and that the conjugate gradient method is fulfilled, since $d_k$ is a slope search direction, and $\propto_k$ is generated from the strong Wolff conditions (SWP) if $\sum_{K=1}^{\infty} \frac{1}{\|d_{k+1}\|^2} = \infty$ then $\lim\limits_{k\to\infty} inf\|g_k\| = 0$.

**Theorem (2):** Suppose that Assumption (1) is fulfilled and that the proposed conjugate gradient method is fulfilled in the direction of the search slope $d_k$, and that the step length $\propto_k$ is generated from the conditions (SWP) then $\lim\limits_{k\to\infty} inf\|g_k\| = 0$.

**Proof:** Using lemma (1), and since the algorithm fulfills theorem (1), and if $g_{k+1} \neq 0$, then we must prove that $\|d_{k+1}\|$ is constrained from above, we take $\|.\|$ for both sides of the Eq. (2) We get

$$\|d_{k+1}\| = \|-g_{k+1} + \beta_k^{New} d_k\| \Rightarrow \|d_{k+1}\| \leq \|g_{k+1}\| + |\beta_k^{New}|\|d_k\|$$

$$|\beta_k^{New}| = \left|\frac{1}{\|g_k\|^2}\left[ g_{k+1}^T\ y_k - \frac{g_{k+1}^T\ d_k}{y_k^T\ d_k}.\ \|y_k\|^2 \right]\right|$$

Using equations (9), (10), (11) we get the following

$$|\beta_k^{New}| \leq \left|\frac{1}{\|g_k\|^2}\left[ 1.2\ \|g_{k+1}\|^2 - \frac{-p\ g_k^T d_k}{-(1-p) g_k^T d_k}.\ \|y_k\|^2 \right]\right|$$

$$|\beta_k^{New}| \leq \left|\frac{1}{\|g_k\|^2}\left[ 1.2\ \|g_{k+1}\|^2 - \frac{p}{(1-p)}.\ \|y_k\|^2 \right]\right| \Rightarrow |\beta_k^{New}| \leq \mathrm{A}$$

$$\|d_{k+1}\| \leq \|g_{k+1}\| + \mathrm{A}\|d_k\| \Rightarrow \|d_{k+1}\| \leq T + \mathrm{AU} \leq \mathrm{M} \Rightarrow \|d_{k+1}\| \leq \mathrm{M} \Rightarrow \frac{1}{\|d_{k+1}\|} \geq \frac{1}{\mathrm{M}}$$

$$\sum_{k=1}^{\infty} \frac{1}{\|d_{k+1}\|^2} \geq \sum_{k=1}^{\infty} \frac{1}{\mathrm{M}^2} = \frac{1}{\mathrm{M}^2}\sum_{k=1}^{\infty} 1 = +\infty$$

According to lemma (1), this leads to

$$\lim_{k\to\infty} inf\|g_k\| = 0.$$

## Sand cat swarm optimization (scso) algorithm

Sand cats are a species of mammals that inhabit harsh desert environments, including the Arabian Peninsula, Central Asia, and the African Sahara. Because of their adaptation, they have thick fur on the palms of their feet, which helps protect the pads from very cold temperatures. Their fur's special qualities also make it challenging to find and follow them [9] and [15].

## Mathematical model for (scso) [16]

The algorithm is designed to decrease this detection threshold, $r_G^{\rightarrow}$, linearly from 2 kilohertz to zero as the algorithm progresses through its iterations.To simulate the exploratory behavior of sand cats, the SCSO algorithm begins with a randomly initialized search space. Each virtual cat in the algorithm is assigned a random initial position, emulating the way sand cats explore new territories in their natural habitat. In this way, cats can explore new areas as described in the equations below:

$$r_G^{\rightarrow} = S_m - \left(\frac{2 \cdot S_m \cdot t}{iter_{max}\ \cdot\ iter_{max}}\right) \tag{12}$$

$$R^{\rightarrow} = 2 \cdot r_G^{\rightarrow} \cdot rand(0,1) - r_G^{\rightarrow} \tag{13}$$

$$r^{\rightarrow} = r_G^{\rightarrow} \cdot rand(0,1) \tag{14}$$

$R^{\rightarrow}$ is the carrier responsible for the conversions and depends on the general sensitivity range $r_G^{\rightarrow}$. Each cat updates its position based on the best candidate position ($pos_{bc}^{\rightarrow}$), its current-position ($pos_c^{\rightarrow}$), and its sensitivity range $r^{\rightarrow}$. Eq.(15) represents the exploration equation.

$$pos^{\rightarrow}(t+1) = r^{\rightarrow}.(\ pos_{bc}^{\rightarrow}(t) - rand(0,1) \cdot pos_c^{\rightarrow}(t)) \tag{15}$$

During the exploitation phase, the methodology for updating the position of each agent 'cat' is based on calculating the distance between the optimal position ( $pos_b^{\rightarrow}$) and its current position ($pos_c^{\rightarrow}$). This process is visualized as navigating the periphery of a circle. Key to this phase is the implementation of a random angle $\theta$, which dictates the direction of each cat's movement. The angle is randomly chosen within a full circular range, translating to values between 0 and 360 degrees. This is mathematically represented as a range between -1 and 1. The SCSO utilizes the roulette wheel selection algorithm to test a random angle and avoid local traps. Incorporating the random angle in

Eq. (17) positively affects the approach of each individual population to hunting and their direction. Equation (17) provides the position update during this phase.

$$\vec{pos}_{rad} = |rand(0,1) \cdot \vec{pos}_b(t) - \vec{pos}_c(t)| \quad (16)$$

$$\vec{pos}(t+1) = \vec{pos}_b(t) - \vec{r} \cdot \vec{pos}_{rad} \cdot cos(\theta) \quad (17)$$

## Whale optimization algorithm (woa)

Whales are predatory animals; they never sleep because whales breathe from the surface of the ocean. In fact, only half of the brain sleeps, and humpback whales are considered intelligent and emotional animals [20]. The most interesting thing about whales is their hunting style. This foraging method is called the bubble-net feeding method [21].

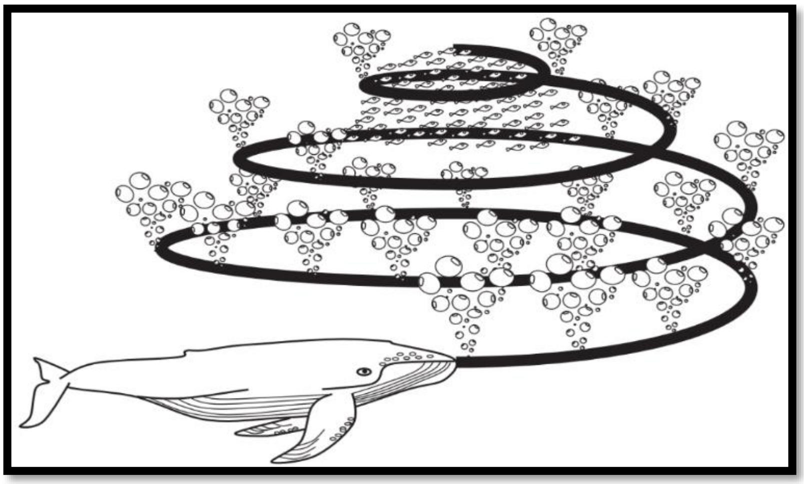


**Fig. 1** *Bubble-net feeding behavior of humpback whales*

## Mathematical model for (woa) [17]

The whale optimization algorithm (WOA) is based on the assumption that the current best candidate solution represents the prey. Once the best search agent is identified, other agents attempt to adjust their positions toward the best location. The following equations explain this:

$$\vec{D} = |\vec{C}.\vec{X^*}(t) - \vec{X}(t)| \quad (18)$$

$$\vec{X}(t+1) = \vec{X^*}(t) - \vec{A}.\vec{D} \quad (19)$$

let t denotes the current iteration and $\vec{A}$, $\vec{C}$ are vectors and $\vec{X^*}$ represents the vector of the best solution location obtained so far. The $\vec{X}$denotes the current position vector. $\vec{X^*}$should be updated in each iteration if a better solution is found. The vectors are calculated as follows:

$$\vec{A} = 2\vec{a}.\vec{r} - \vec{a} \quad (20)$$

$$\vec{C} = 2\vec{r} \quad (21)$$

Where $\vec{a}$ decreases linearly from 2 to 0 over iterations during the exploration and exploitation phases, while $\vec{r}$ is a random vector expression ranging from 0 to 1. Whales use a bubble network strategy when attacking prey, and this can be mathematically described as follows:

## Bubble network attack method (exploitation phase)

In order to develop a mathematical model of the bubble network behavior of humpback whales, two approaches were designed as follows:

**1- Deflationary enclosure mechanism:** This behavior is achieved by reducing the value of $\vec{a}$in the equation. (20). Which leads to a decrease in the value of the vector $\vec{A}$. Determine random values for $\vec{A}$ between [-1,1]. The search agent's new location can be specified anywhere between the search agent's original location and the current best agent's location.

**2- Spiral update location:** There is a spiral equation that was created between the location of the whale and the prey to imitate the spiral-shaped movement of humpback whales, as shown in Eq. (22).

$$\vec{X}(t+1) = \vec{D'} . e^{bl} . \cos(2\pi l) + \vec{X^*}(t) \tag{22}$$

So that $\vec{D'} = |\vec{X^*}(t) - \vec{X}(t)|$ indicates the distance between the whale and the prey. b is to stir in a spiral manner. l indicates a random number in [-1,1]. Humpback whales have been observed to swim around the prey in a contracting circle and along a spiral path simultaneously.

$$\vec{X}(t+1) = \begin{cases} \vec{X^*}(t) - \vec{A}.\vec{D} & if\ \ p < 0.5 \\ \vec{D'}.e^{bl}.\cos(2\pi l) + \vec{X^*}(t) & if\ \ p \geq 0.5 \end{cases} \tag{23}$$

**Searching for prey (expioration phase)**

The search agent's location is updated in the exploration phase based on a randomly selected search agent rather than the best search agent obtained so far. When A>1, this process confirms exploration and allows the WOA algorithm to perform a global search. The following mathematical model demonstrates this:

$$\vec{D} = |\vec{C}.\vec{X}_{rand} - \vec{X}| \tag{24}$$

$$\vec{X}(t+1) = \vec{X}_{rand} - \vec{A}.\vec{D} \tag{25}$$

where $\vec{X}_{rand}$ is a random location vector. To model this behavior, the choice of a search agent can be incorporated randomly when |A|>1, and choose the best solution when |A|<1. In addition, depending on the P value, the WOA algorithm can switch between spiral or circular motion. Finally, the WOA algorithm terminates when the completion criterion is met.

## Improve the whale optimization algorithm (woa)

Metaheuristic algorithms are a modern and innovative approach to artificial intelligence. Swarm-based algorithms are an effective tool for solving complex problems and achieving common goals through continuous interaction and collaboration among individual members.

In this section, the two proposed algorithms will be presented (CG-WOA) and (SCSO-WOA).

## Improve the whale optimization algorithm (woa) by developed conjugate gradient algorithm

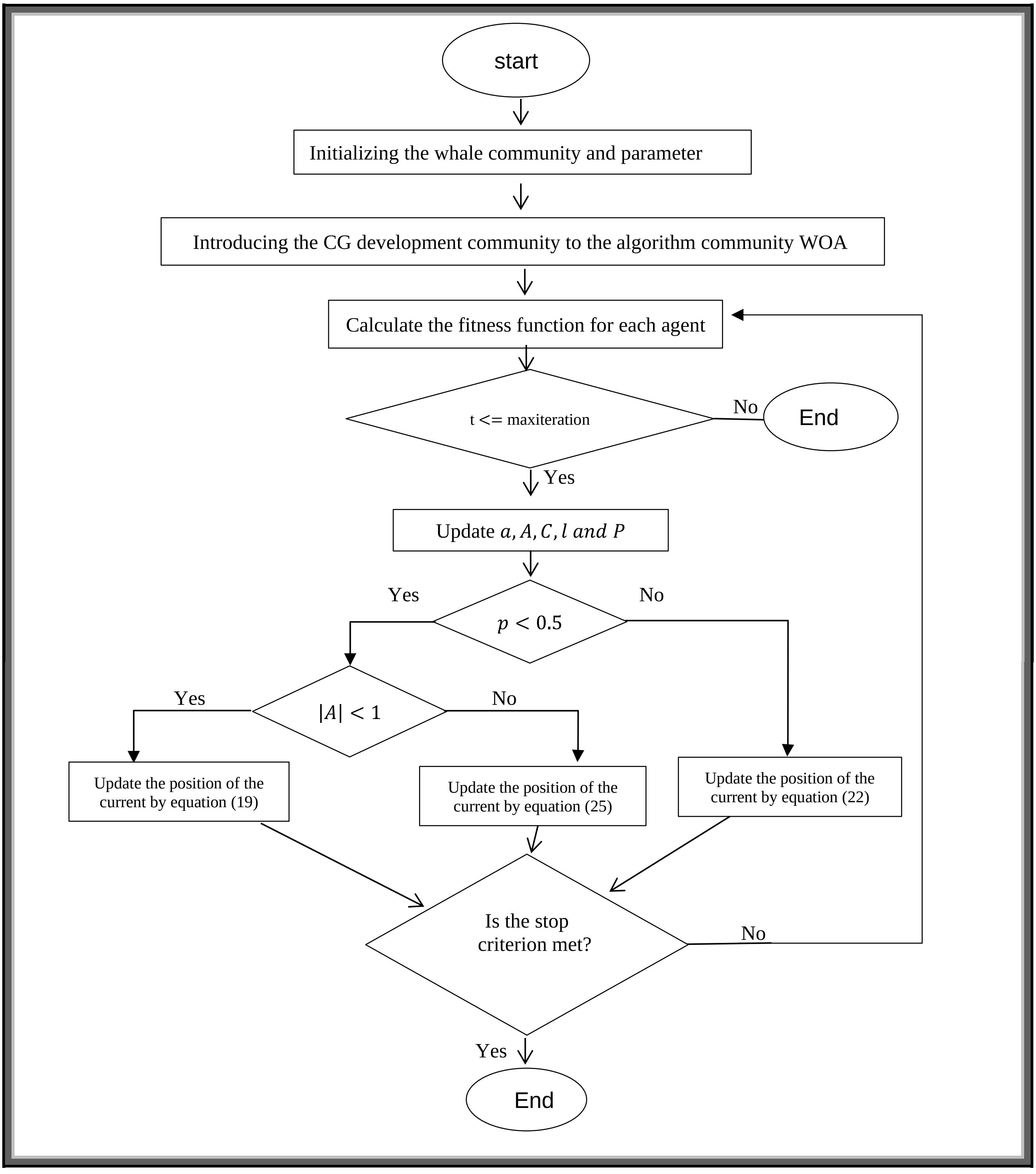
The conjugate gradient is an iterative method that starts from a specific point in a direction and generates a sequence of iterations until the minimum value of the function is reached. The hybridization process occurs by linking two methods, such as if, for example, one of the two methods has good computational properties and the second method has strong comprehensive convergence properties.

In the second section of this research, a new parameter was derived for the purpose of calculating the new direction. We use the equation (7).

**Algorithm of (CG -WOA)**

1 start and initializing the whale community and parameter
2- Introducing the CG development community to the algorithm community WOA
3-Calculate the fitness of each search agent.
4- $X^*$the best search agent.
5-**While** (t <= maximum number of iteration)
6- **For** each search agent
7- $\mathbf{IF_1}(p < 0.5)$
8- $\mathbf{IF_2}(|A| < 1)$
9- Update the position of the current search agent by equation (1).
10- **Else** $\mathbf{IF_2}$ $(|A| \geq 1)$

Select a random search agent ($X_{rand}$).
11- Update the position of the current search agent by equation (8).
12- **End IF$_2$**
13- **Else IF$_1$** ($p \geq 0.5$)
14- Update the position of the current by equation (5)
15- **End IF$_1$**
16- **End for**
20- t=t+1
21-**End While**




**FLOWCHART 1**. *The steps of the CG-WOA*

**Improve algorith of sand cat swarm optimization (scso) with whale optimization algorithm (woa)**

The Whale Optimization Algorithm (WOA) was Improve with the Sand Cat Swarm Optimization Algorithm (SCSO). So, the exploitation equation in the Whale Optimization Algorithm was replaced with the exploitation equation in the Sand Cat Swarm Optimization algorithm after improving it in the proposed algorithm (SCSO-WOA) by adding some attributes. desired results in both algorithms, and good numerical results were obtained compared with the results of the original algorithm. We will explain this in detail in the following Improve mathematical model:

**Mathematical model for (scso-woa)**

The search space will be randomly selected such that the search agents' locations are based on random locations. The prey search method will be used in the SCSO algorithm, which simulates the prey search behavior of sand cats in nature, which is represented by the emission of low-frequency noise. The sensitivity range will be determined for each search agent. The proposed algorithm (SCSO-WOA) in moving through the exploration and exploitation stages depends on the value of $A^{\rightarrow}$shown in the following equation (26):

$$A^{\rightarrow} = 2a^{\rightarrow}.r^{\rightarrow} - a^{\rightarrow} \tag{26}$$

**Expioration phase**

The exploration phase begins if |A|≥1, so that the search agents explore new areas in search of prey randomly based on the value of p, where p is a random vector between [0,1] so that a random location is chosen and the distance between the current location and a random location is calculated. It was chosen randomly as shown in equation (27) and as we mentioned previously $X^{\rightarrow}_{rand}$ is a random location vector. At each iteration, the search agents update their locations according to the following equation (28):

$$\mathrm{D}^{\rightarrow} = |\mathrm{C}^{\rightarrow}.X^{\rightarrow}_{rand} - X^{\rightarrow}| \tag{27}$$

$$\mathrm{X}^{\rightarrow}(t+1) = X^{\rightarrow}_{rand} - A^{\rightarrow}.\mathrm{D}^{\rightarrow} \tag{28}$$

**Exploitation phase**

The exploitation phase begins if |A|<1. The exploitation phase is made use of by the Sand Cat Swarm Optimization (SCSO) algorithm, which mimics the behavior of sand cats in nature. The search agents update their positions between the best search agent obtained so far and the original location, so that it is first calculated the distance vector between the best location multiplied by a random value confined to the interval [0,1] and the original location multiplied by the sensitivity of the search agent as shown by the following equations:

$$r^{\rightarrow} = r^{\rightarrow}_{G} * rand(0,1) \tag{29}$$

$$D_{leader} = |rand(0,1) * \mathrm{X}^{*\rightarrow}(\mathrm{t}) - r * X^{\rightarrow}(t)| \tag{30}$$

At each iteration, the search agents update their locations based on the p values, as in equation (31):

$$\mathrm{X}^{\rightarrow}(t+1) = \begin{cases} A^{\rightarrow} * D_{leader} + r * \mathrm{X}^{*\rightarrow}(\mathrm{t}) * \cos(\theta) & if \ \ p < 0.5 \\ \\ A^{\rightarrow} * D_{leader} - r * \mathrm{X}^{*\rightarrow}(\mathrm{t}) * \cos(\theta) & if \ \ p \geq 0.5 \end{cases} \tag{31}$$

So, the direction of motion is determined by a random angle (θ) on the circle. Because the chosen random angle ranges between 0 and 360, meaning its value will be between -1 and 1. The proposed algorithm (SCSO-WOA) takes advantage of the roulette wheel selection algorithm to test a random angle as well as to avoid falling into the local trap. Using the random angle in Equation (31) will have a positive effect on the approach of each individual population to hunting and their direction. Table (1) shows the standard test functions. Scheme (2) shows the steps of the algorithm.

**Algorithm of (scso-woa)**

1-Initialize the population.

2-Calculate the fitness function of the objective function.

3- $X^*$the best search agent.

4- Initialize the $r, r_G, A, C, p$.

5-**While** (t <= maximum iteration)

6- **For** each search agent.

7- $\mathbf{IF_1}(p < 0.5)$

8- $\mathbf{IF_2}$ $(|A|<=1)$ then

9- Update the search agent position based on the eq.31: $X^{\rightarrow}(t+1) = A^{\rightarrow} * D_{leader} + r * X^{*\rightarrow}(t) * \cos(\theta)$ .

10- **Else** $\mathbf{IF_2}$ $(|A| \geq 1)$

11- Update the search agent position based on the Eq.28: $X^{\rightarrow}(t+1) = X^{\rightarrow}_{rand} - A^{\rightarrow}.D^{\rightarrow}$.

12- **End** $\mathbf{IF_2}$

13- **Else** $\mathbf{IF_1}$ $(p \geq 0.5)$

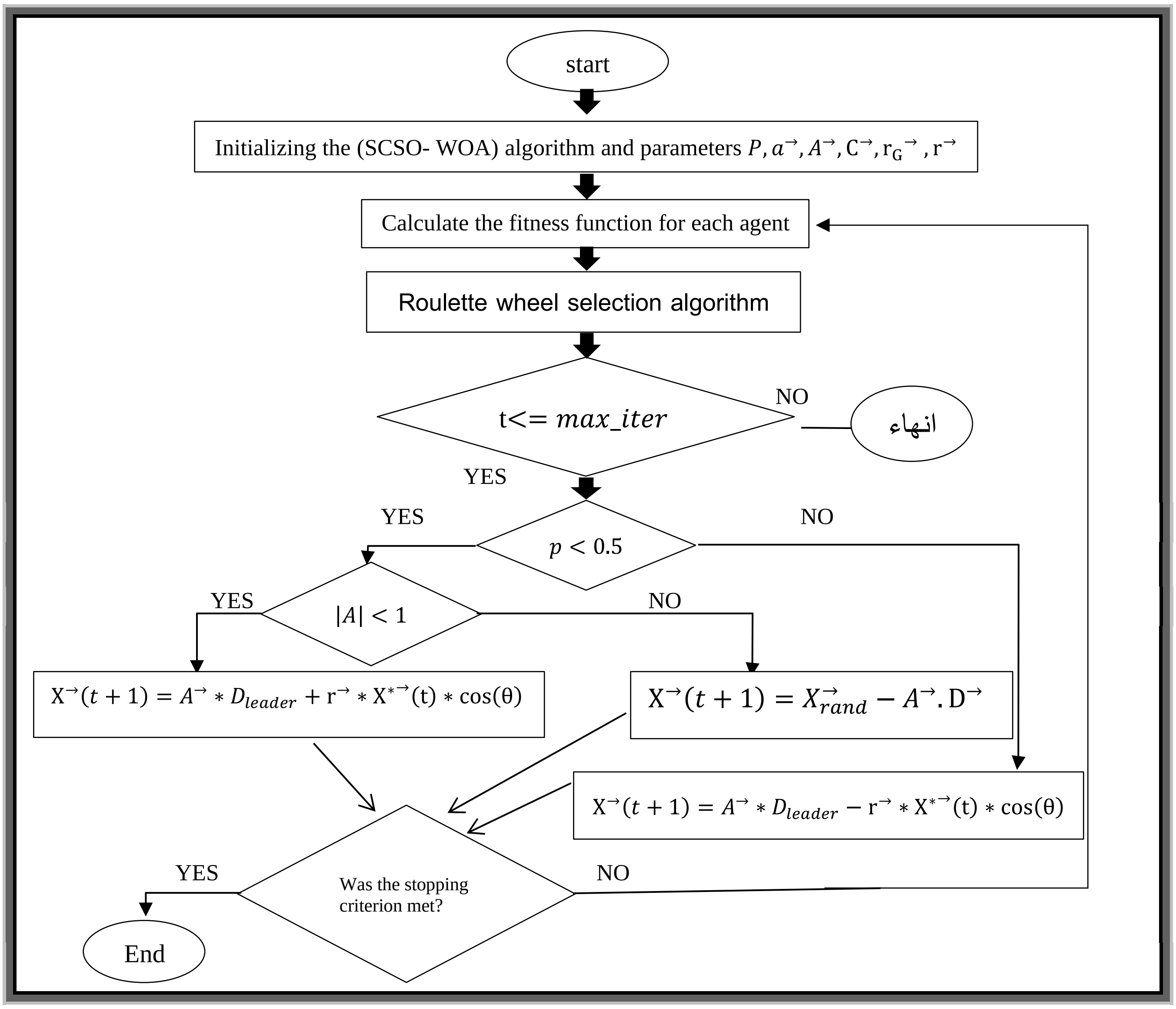
14- eq.31: $X^{\rightarrow}(t+1) = A^{\rightarrow} * D_{leader} - r * X^{*\rightarrow}(t) * \cos(\theta)$ .

15- **End** $\mathbf{IF_1}$

16- **End for**

17- t=t+1

13-**End**




**FLOWCHART 2**. *The steps of the SCSO- WOA*

**TABLE 1:** Introducing standard benchmark test function (Unimodal, Multimodal) that is used to assess the efficiency of algorithms.[16]

| Function Symbol | Function Name | Function | D | Range | $f_{min}$ |
|---|---|---|---|---|---|
| $F_1$ | Sphere | $\sum_{i=1}^{n} x_i^2$ | 30 | $[-100, 100]$ | 0 |
| $F_2$ | Schwefel (2.22) | $\sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$ | 30 | $[-10, 10]$ | 0 |
| $F_3$ | Schwefel (1.2) | $\sum_{i=1}^{n} \left( \sum_{j-1}^{i} x_j \right)^2$ | 30 | $[-100, 100]$ | 0 |
| $F_4$ | Schwefel (2.21) | $\max_i \{ \lvert x_i \rvert , 1 \leq i \leq n \}$ | 30 | $[-100, 100]$ | 0 |
| $F_5$ | Rastrigin | $\sum_{i=1}^{n} \left[ x_i^2 - 10 \cos(2\pi x_i) + 10 \right]$ | 30 | $[-5 \cdot 12, 5 \cdot 12]$ | 0 |

**TABLE 2:** Comparison outcomes of SCSO, WOA, CG- WOA and SCSO- WOA using the number of elements along with 30 elements and the number of Iterations 500.

| Function Symbol | SCSO | WOA | OCG-WOA | SCSO- WOA |
|---|---|---|---|---|
| $F_1$ | 2.2387e-118 | 2.22e-81 | 2.0676e-318 | 0 |
| $F_2$ | 1.676e-62 | 1.7725e-53 | 3.5535e-162 | 0 |
| $F_3$ | 3.5931e-104 | 47219.6092 | 3.0956e-283 | 0 |
| $F_4$ | 1.7584e-53 | 73.6108 | 2.8945e-153 | 0 |
| $F_5$ | 0 | 0 | 0 | 0 |

**TABLE 3:** Comparison outcomes of SCSO, WOA, CG- WOA and SCSO-WOA using the number of elements along with 60 elements and the number of Iterations 500.

| Function Symbol | SCSO | WOA | OCG-WOA | SCSO- WOA |
|---|---|---|---|---|
| **$F_1$** | **8.724e-120** | **3.2966e-95** | **0** | **0** |
| **$F_2$** | **3.8765e-66** | **2.0731e-56** | **2.7024e-168** | **0** |
| **$F_3$** | **1.3968e-109** | **29322.0197** | **3.5229e-287** | **0** |
| **$F_4$** | **1.2347e-56** | **18.3039** | **1.3678e-158** | **0** |
| **$F_5$** | **0** | **0** | **0** | **0** |

**Analysis of results**

In general, the above tables showed the success of the two proposed algorithms (SCSO-WOA) and (CG-WOA) in obtaining the optimal solution for five standard test functions, which are (0). Figures (2,3,4,5,6) showed the efficiency of the two proposed algorithms and also showed the curve of approaching the optimal solution. The red color indicates the curve of the proposed algorithm (SCCO-WOA), the cyan color indicates the curve of the proposed algorithm (CG -WOA), the green color indicates the curve of (WOA) Algorithm. In contrast, the blue color indicates the curve of the (SCSO) Algorithm.

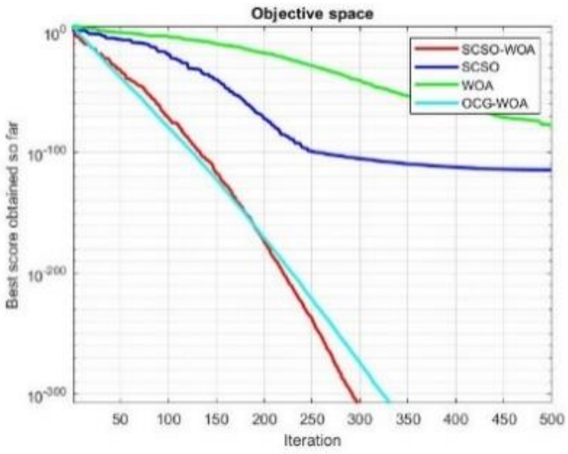



**Fig. 2** *function graph for $F_1$*

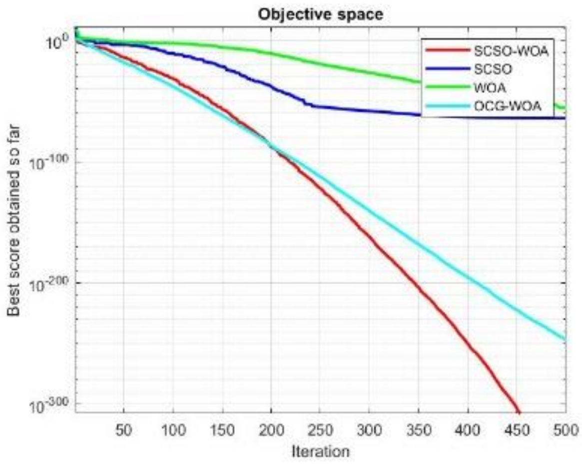



**Fig. 3** *function graph for $F_2$*

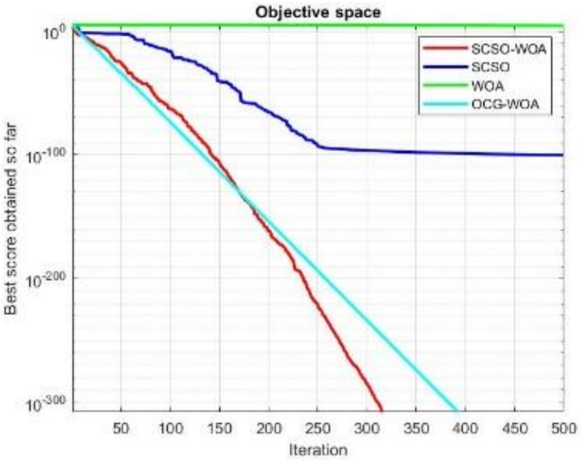



**Fig. 4** *function graph for* $F_3$

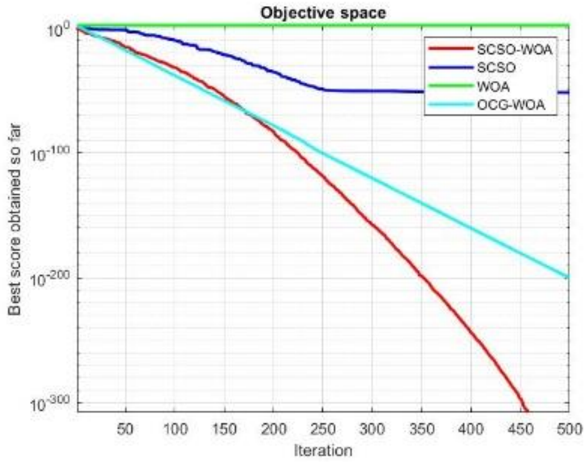



**Fig. 5** *function graph for* $F_4$

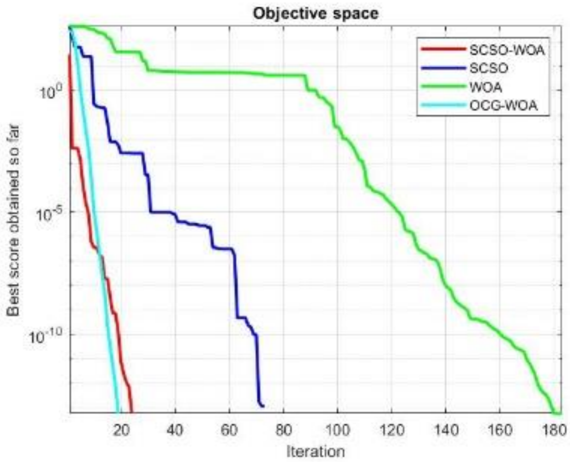



**Fig. 6** *function graph for* $F_5$

## Conclusions

The Improve of the Whale Algorithm (WOA) has given rise to two new proposed algorithms (SCSO-WOA) and (CG-WOA). These two algorithms have several properties, including their ability to work with complex, multi-dimensional problems. And the efficiency of their speed compared to the speed of the two original algorithms, so that the two proposed algorithms (SCSO-WOA) and (CG-WOA) gave better numerical results than the results of the two original algorithms with less time and effort, as can be seen from Tables (2) and (3) above for five of the standard test functions. Which shows comparative results with 500 iterations and 30 and 60 search items, respectively.

Through these features, the two proposed algorithms (SCSO-WOA) and (CG-WOA) can be exploited to solve many problems, including improving performance, reducing cost, and improving engineering and mathematical design.

## References


1. Li, H., Liu, X., Huang, Z., Zeng, C., Zou, P., Chu, Z., & Yi, J. (2020). Newly emerging nature-inspired optimization-algorithm review, unified framework, evaluation, and behavioural parameter optimization. IEEE Access, 8, 72620-72649.
2. SS, V. C., & HS, A. (2022). Nature inspired meta heuristic algorithms for optimization problems. Computing, 104(2), 251-269.
3. Daneshmand, A., Facchinei, F., Kungurtsev, V., & Scutari, G. (2015). Hybrid random/deterministic parallel algorithms for convex and nonconvex big data optimization. IEEE Transactions on Signal Processing, 63(15), 3914-3929.
4. Del Ser, J., Osaba, E., Molina, D., Yang, X. S., Salcedo-Sanz, S., Camacho, D., ... & Herrera, F. (2019). Bio-inspired computation: Where we stand and what's next. Swarm and Evolutionary Computation, 48, 220-250.
5. Hopper, E. B. C. H., & Turton, B. C. (2001). An empirical investigation of meta-heuristic and heuristic algorithms for a 2D packing problem. European Journal of Operational Research, 128(1), 34-57.
6. Tanabe, R., & Ishibuchi, H. (2019). A review of evolutionary multimodal multiobjective optimization. IEEE Transactions on Evolutionary Computation, 24(1), 193-200.
7. Khattab, H., Sharieh, A., & Mahafzah, B. A. (2019). Most valuable player algorithm for solving minimum vertex cover problem. International Journal of Advanced Computer Science and Applications, 10(8):167-159
8. Houssein, E. H., Gad, A. G., Wazery, Y. M., & Suganthan, P. N. (2021). Task scheduling in cloud computing based on meta-heuristics: review, taxonomy, open challenges, and future trends. Swarm and Evolutionary Computation, 62, 100841.
9. Masadeh, R., Alsharman, N., Sharieh, A., Mahafzah, B. A., & Abdulrahman, A. (2021). Task scheduling on cloud computing based on sea lion optimization algorithm. International Journal of Web Information Systems, 17(2), 99-116.
10. Mahafzah, B. A., Jabri, R., & Murad, O. (2021). Multithreaded scheduling for program segments based on chemical reaction optimizer. Soft Computing, 25, 2741-2766.
11. Singh, P., Chaudhury, S., & Panigrahi, B. K. (2021). Hybrid MPSO-CNN: Multi-level particle swarm optimized hyperparameters of convolutional neural network. Swarm and Evolutionary Computation, 63, 100863.
12. Zhao, D., Liu, L., Yu, F., Heidari, A. A., Wang, M., Oliva, D., ... & Chen, H. (2021). Ant colony optimization with horizontal and vertical crossover search: Fundamental visions for multi-threshold image segmentation. Expert Systems with Applications, 167, 114122.
13. Gheisarnejad, M. (2018). An effective hybrid harmony search and cuckoo optimization algorithm based fuzzy PID controller for load frequency control. Applied Soft Computing, 65, 121-138.
14. Li, G., & Hu, H. (2014). Risk design optimization using many-objective evolutionary algorithm with application to performance-based wind engineering of tall buildings. Structural Safety, 48, 1-14.

15. Kaveh, A. (2017). Applications of metaheuristic optimization algorithms in civil engineering. Basel, Switzerland: Springer International Publishing.

16. Seyyedabbasi, A., & Kiani, F. (2023). Sand Cat swarm optimization: A nature-inspired algorithm to solve global optimization problems. Engineering with Computers, 39(4), 2627-2651.

17. Mirjalili, S., & Lewis, A. (2016). The whale optimization algorithm. Advances in engineering software, 95, 51-67.

18. Beale, E. M. L. (Ed.). (1988). *Introduction to optimization*. Wiley-Interscience.

19. Zhang, L., Zhou, W., & Li, D. H. (2006). A descent modified Polak–Ribière–Polyak conjugate gradient method and its global convergence. IMA Journal of Numerical Analysis, 26(4), 629-640.

20. Hof, P. R., & Van der Gucht, E. (2007). Structure of the cerebral cortex of the humpback whale, Megaptera novaeangliae (Cetacea, Mysticeti, Balaenopteridae). The Anatomical Record: Advances in Integrative Anatomy and Evolutionary Biology: Advances in Integrative Anatomy and Evolutionary Biology, 290(1), 1-31.

21. Goldbogen, J. A., Friedlaender, A. S., Calambokidis, J., Mckenna, M. F., Simon, M., & Nowacek, D. P. (2013). Integrative approaches to the study of baleen whale diving behavior, feeding performance, and foraging ecology. BioScience, 63(2), 90-100.

IRAQI Academic Scientific Journals

Samarra Journal of Pure and Applied Science

SJPAS
مجلة سامراء للعلوم الصرفة والتطبيقية

www.sjpas.com

p ISSN: 2663-7405
e ISSN: 2789-6838

# استخدام خوارزمية التدرج المترافق المطورة وخوارزمية أمثلة سرب القطط الرملية (SCSO) لتحسين أداء خوارزمية أمثلة الحوت (WOA)

**عمر ظاهر شلال [1,2]*، بان احمد [1]**

1- قسم الرياضيات، كلية علوم الحاسوب والرياضيات، جامعة الموصل، العراق

2- المديرية العامة للتربية في محافظة نينوى، وزارة التربية، العراق

البحث مستل من اطروحة دكتوراه الباحث الاول

**معلومات البحث:**

تأريخ الاستلام: 2024/04/28

تاريخ التعديل : 2024/05/31

تأريخ القبـــول: 2024/06/02

تاريخ الــنشر: 2024/12/30

**الكلمات المفتاحية:**

*الخوارزميات الحدسية، خوارزمية التدرج المترافق، خوارزمية أمثلة سرب القطط الرملية (SCSO)، خوارزمية تحسين الحوت (WOA)، الخوارزميات الهجينة.*

**معلومات المؤلف**

الايميل:

الموبايل:+9647716895052

**الخلاصة:**

في هذا البحث، تم استخدام استراتيجيتين متميزتين لتعزيز قدرات حل المشكلات. تضمنت الإستراتيجية الأولى تطوير خوارزمية التدرج المترافق بحيث تم اشتقاق واقتراح معلمة جديدة. وتضمنت الاستراتيجية الثانية تهجين خوارزمية أمثلة الحوت (WOA) بطريقتين، الأولى باستخدام المجتمع من خلال الاستفادة من خوارزمية التدرج المترافق المطورة والتي تم استخراجها من الاستراتيجية الأولى والحصول على خوارزمية هجينة (CG-WOA) تعطي نتائج أفضل من نتائج الخوارزمية الأصلية. الطريقة الثانية هي الدمج بين خوارزمية أمثلة سرب القطط الرملية (SCSO) مع خوارزمية أمثلة الحيتان (WOA)، ويتم الحصول على خوارزمية هجينة (SCSO-WOA). تتمتع الخوارزميتان المقترحتان (CG-WOA) و (SCSO-WOA) بالعديد من الخصائص، بما في ذلك قدرتها على التعامل مع مشاكل التحسين المعقدة، وكفاءة سرعتها مقارنة بالخوارزمية الأصلية. إن تنوع عمليات الاستكشاف والاستغلال في الخوارزمية الهجينة (SCSO-WOA) يمنحها ميزة التقارب السريع والحصول على الحل الأمثل العالمي وتجنب الوقوع في الحلول المحلية. تم اختبار كفاءة الخوارزميتان (CG-WOA) و(SCSO-WOA) على خمس دوال من دوال اختبارية القياسية، بحيث تم الحصول على نتائج عددية أفضل من نتائج الخوارزمية الأصلية.