

## Some Properties On $pre-\nu$ -open sets

Inam Owaid Khudhair\*, and Luma Saad Abdulbaqi

Mathematic department, College of Education for Women, Tikrit University, Iraq



This work is licensed under a [Creative Commons Attribution 4.0 International License](https://creativecommons.org/licenses/by/4.0/)

<https://doi.org/10.54153/sjpas.2024.v6i4.629>

### Article Information

Received: 10/07/2023

Revised: 31/07/2023

Accepted: 04/08/2023

Published: 30/12/2024

### Keywords:

$pre$ -open,  $pre-\nu oX$ ,  
 $pre\ c(\nu oX)$ ,  $per\ o(\nu oX)$ ,  
 $pre-\nu$ -open,  $\beta\nu$ -open,  
 $beta(\nu oX)$ ,  $\nu$ -exterior, and  
 $pre(\beta)$ - $\nu$ -open.

### Corresponding Author

E-mail:

[inam.owaid379@st.tu.edu.iq](mailto:inam.owaid379@st.tu.edu.iq)

Mobile: 07703473484

### Abstract

Various definitions of the  $pre-\nu$ -open set, also known as a  $\nu$ -open set, are described in this article. The purpose of this work is to investigate the  $pre-\nu$ -open set's characteristics, as well as how they relate to other types of open sets. Additionally, there is a connection between the open set in a  $\nu$ -open set in the universal set  $X$  and the open set.,  $pre$ -open set, as well as their opposite via instances. The study examines the relationship between the type  $pre-\nu$ -open set,  $\nu$ -open set,  $pr$ -open set, and the intersection of  $\tau_k$  for all  $k$  in  $J$ . It also investigates the qualities and essential theorems, as well as the criteria for the new types of  $pr-\nu$ -open set. This study focuses on analyzing the properties of  $pr_{\nu}-\beta_{\nu}$ -open sets and their relationship with  $pr_{\nu}$ -open,  $\nu$ -open sets, and the intersection of  $\tau_k$  for all  $k$  in  $J$ . Additionally, examples will be used to illustrate the construction of their converse.

### Introduction:

Many researchers presented studies on the subject of our study and made generalizations regarding this study, and we mention them, for example, but not limited to, this survey article will cover the majority of current  $pre$ -open sets research as a particular sort of generalized topology (top.) on  $X$ , Mooakher et al. present a technique to extract the collection  $PO(X)$  of all  $pr$ -open sets of the top. space  $(X)$  from  $\tau[2]$ , Sathishmohan et al. explore the characteristics of nano- $pre$  setups. Additionally, we have obtained some of their fundamental findings and provided pertinent examples to help people comprehend abstract ideas. [6], the researcher's Jun et al. in 2008 presented an applied study about  $pre$ -open sets [3]; supra  $pre$ -open sets and supra  $pre$ -continuous maps are a new type of sets and maps, respectively, that belong to the category of the top. spaces, are developed and researched, it was Sayed in a 2010 study [1], In tri top. spaces, Tapi et al. offer two new forms of open sets, namely tri semi-open sets and tri  $pre$ -open sets, together with their various features and characterization [4], in 2017 introduce Rao and Srinivasa paper goal is to expand on some other top.-related subjects in neutrosophic top.[5].

The researcher Mashhour presented open sets of the  $pr$  type for the first time in 1983 [8], after that's this type of sets was studied and expanded in studies, Subsequently the researcher Zaman presented in 2022 the study of  $\nu$ -space. As for what we are about to do, it is to form a

merger between them and explain the properties of the resulting sets, that's we will call  $pr_v$ -open.

## 1- Preliminaries

**1.1.Definition:** [9] A subset  $A$  of a space is considered to be a pre-open set if  $\mathcal{A}$  is contained within the interior of the closure of  $A$ . The complement of a pre-open set is considered to be a pre-closed set.

**1.2.Definition:** [9] In the context of top. spaces  $(X, \tau)$ , a point  $a \in \mathcal{A}$  is referred to as The interior of set  $\mathcal{A}$  in  $X$  is defined as the set of all elements an in  $\mathcal{A}$  for that's there exists an open set  $U_x$  in the family  $\tau$  such that an is an element of  $U_x$  and  $U_x$  is a subset of  $\mathcal{A}$ . In other words, the interior of  $A$  is the union of all open sets that contain  $\mathcal{A}$ , denoted as  $\mathcal{A}^\circ$  equal union The set  $O$  is an open set and it is a subset of  $\mathcal{A}$ .

**1.3.Definition:** [8] Consider a top. space  $(X, \tau)$  and let  $\mathcal{A}$  be a subset of  $X$ . The border of set  $\mathcal{A}$  is defined as the collection of points that are neither part of the interior nor the exterior of  $A$ . It is denoted by  $\mathcal{A}^b$  or  $b(\mathcal{A})$ ,  $b(\mathcal{A}) = X / (\mathcal{A}^\circ \cup \text{ext}(\mathcal{A}))$ .

**1.4.Definition:**[7] Consider a collection  $\{\tau_k\}_{2 \leq k \in J}$  of open sets in the top. space  $X$ . If there are no two non-empty sets that do not overlap in the intersection of all sets  $\tau_k$  for  $k$  in  $J$ . A  $v$ -space  $(X, v\circ X)$  satisfies the criteria of top. on  $X$ .

**1.5.Definition:**[7] The definition of  $v\circ X$ , that's stands for "set of all  $v$ -open sets in  $X$ ," is  $v\circ X = \{x: x \text{ is a } v\text{-open set in } X\}$ .

**1.6.Definition:**[7] A  $v$ -closed set is the set that is the complement of a  $v$ -open set.  $CS_v$  is the index of the collection of all  $v$ -closed sets.

**1.7.Definition:**[7] An order pair  $(X, v\circ X)$  is called  $v$ -space where  $X$  is a universal set and  $v\circ X$  is set of all  $v$ -open in  $X$ .

A pair  $(X, v\circ X)$  is referred to as a  $v$ -space, where  $X$  represents a universal set and  $v\circ X$  represents the set of all  $v$ -open sets in  $X$ .

**1.8.Definition:**[7] Consider a vector space  $(X, v\circ X)$  and let  $\mathcal{A}$  be a subset of  $X$ . A point  $b \in \mathcal{A}$  is considered to be the  $v$ -interior of  $\mathcal{A}$  if and only if there exists a set  $x \in v\circ X$  such that  $b \in x \subseteq \mathcal{A}$ .

**1.9.Definition:**[7] Consider a vector space  $(X, v\circ X)$  and let  $A$  be a subset of  $X$ . The collection of all  $v$ -interior points of set  $\mathcal{A}$  is referred to as the  $v$ -interior of  $\mathcal{A}$  and symbolized as  $\text{int}_v(A)$ .

## 2- $v$ -open sets

**2.1. Definition:** Take  $(X, v\circ X)$  be an top. space  $G_n$  and  $\kappa$  is  $v$ -open set in  $X$  Subsequently  $\kappa$  is called pre- $v$ -open set if  $[\kappa \subseteq \text{int}(\text{cl}(\kappa))]$ , the complement of pre-  $v\circ X$  set is allegedly is allegedly

pre  $U \subset X$ , the collection of all pre  $v\circ X$  (respect pre -  $v\circ X$ ) subset of  $(X, v\circ X)$  is denoted by per  $o(v\circ X)$  (respect pre  $c(v\circ X)$ ).

**2.2. Example:** Let  $X = \{6,7,8\}$

$$\tau_1 = \{X, \emptyset, \{6\}, \{7\}, \{6,7\}, \{6,8\}\}, \tau_2 = \{X, \emptyset, \{6\}, \{6,7\}\}$$

$$\tau_1 \cap \tau_2 = \{X, \emptyset, \{6\}, \{6,7\}\}$$

$$v = \{X, \emptyset, \{6\}, \{6,7\}, \{6,8\}\}, v^c = \{X, \emptyset, \{6\}, \{6,7\}, \{6,8\}\}$$

$$pr_v\text{-open set } \kappa \subseteq \text{int}(\text{cl}(\kappa))$$

$$\text{Subsequently } pr_v\text{-open} = \{\{6\}, \{6,8\}, \{6,7\}, X, \emptyset\}$$

**2.3.Remark:** Let  $\{\tau_k\}_{k \in J}$  be a family of top. space on  $X$  and  $N \in \tau_k$  Subsequently  $\kappa \in pr_{\nu}$ -open, Subsequently  $N \in pr_{\nu}$ -open. ( $pr_{\nu}$ -open  $\Rightarrow \beta\nu$ -open).

Proof : Consider an arbitrary open set  $N$

$$\text{Since } N \subseteq \text{cl}(N) \Rightarrow \text{Subsequently } N = \text{int}(N) \subseteq \text{int}(\text{cl}(N))$$

$$N \subseteq \text{int}(\text{cl}(N)) \Rightarrow \text{cl}(N) \subseteq \text{cl}(\text{int}(\text{cl}(N))) \Rightarrow N \subseteq \text{cl}(\text{int}(\text{cl}(N)))$$

**2.4.Remark:** Consider a family  $\{\tau_k\}_{k \in J}$  of top. spaces on  $X$ . Let  $x$  be an element of  $\tau_k$  such that  $x$  is  $pr$ - $\nu$ -open. Subsequently,  $x$  is also  $pr$ - $\nu$ -open.

**2.5. Remark:** Let  $\{\tau_k\}_{k \in J}$  be a family of top. space on  $X$  and  $\kappa \in \tau_k$  Subsequently  $\kappa \in pr_{\nu}$ -open for all  $k \in J$ .

Consider the family  $\{\tau_k\}_{k \in J}$  of top. spaces on  $X$ . Let  $x \in \tau_k$ . It follows that  $x \in pre_{\nu}$ -open for all  $k \in J$ .

Proof : Consider an arbitrary open set  $\mathcal{A}$

$$\text{Since } \mathcal{A} \subseteq \text{cl}(\mathcal{A}) \Rightarrow \text{Subsequently } \mathcal{A} = \text{int}(\mathcal{A}) \subseteq \text{int}(\text{cl}(\mathcal{A}))$$

$$\mathcal{A} \subseteq \text{int}(\text{cl}(\mathcal{A})) \Rightarrow \text{cl}(\mathcal{A}) \subseteq \text{cl}(\text{int}(\text{cl}(\mathcal{A}))) \Rightarrow \tau \subseteq \text{cl}(\text{int}(\text{cl}(\tau)))$$

**2.6. Remark:** The intersection of two sets of  $pr_{\nu}$  is not necessarily belong to  $pr_{\nu}$ -open.

**2.7. Example:** Let  $X = \{6,7,8,9\}$

$$\tau_1 = \{X, \emptyset, \{7,8,9\}, \{8,9\}, \{8\}\}, \quad \tau_2 = \{X, \emptyset, \{7,8,9\}, \{8,9\}, \{9\}\}$$

$$\text{Subsequently } \nu o X = \{X, \emptyset, \{8,9\}, \{6,8,9\}, \{7,8,9\}\}, \nu o X^c = \{\{6,7\}, \{7\}, \{6\}, X, \emptyset\}$$

$$\text{Thus } pr_{\nu}(X) = \left\{ \begin{array}{l} \{\emptyset, X, \{8\}, \{9\}, \{6,8\}, \{6,9\}, \{7,8\}, \{7,9\}, \{8,9\}\} \\ \{\{6,7,8\}, \{6,7,9\}, \{6,8,9\}, \{7,8,9\}\} \end{array} \right\}$$

It is clear that  $\{6,8\} \cap \{6,9\} = \{6\}$  and  $\{6\} \notin pr_{\nu}$ -open

Also  $\{6,7,8\} \cap \{6,7,9\} = \{6,7\}$  and  $\{6,7\} \notin pr_{\nu}$ -open

**2.8.Note:** Every open set is also  $pr$ -open, although this is not true in general for quench.

Proof : Consider an arbitrary open set  $\mathcal{A}$

$$\text{Subsequently } \mathcal{A} = \text{int}(\mathcal{A}), \quad \text{Considering } \mathcal{A} \subseteq \text{cl}(\mathcal{A}) \Rightarrow \mathcal{A} = \text{int}(\mathcal{A}) \subseteq \text{int}(\text{cl}(\mathcal{A}))$$

$$\mathcal{A} \subseteq \text{int}(\text{cl}(\mathcal{A}))$$

**2.9.Note:** Let  $\{\tau_k\}_{k \in J}$  be a family of top. space on  $X$ ,  $\mathcal{T} \in \tau_k$  for all  $k \in J$ , if  $\mathcal{T}$  is pre-open not need to belong to top..

**2.10. Example:** Let  $X = \{6,7,8,9\}$

$$\tau_1 = \{X, \emptyset, \{6,8\}, \{6,9\}, \{6\}, \{6,8,9\}\}, \tau_2 = \{X, \emptyset, \{8,9\}, \{6,7,9\}, \{9\}\}$$

$$\text{Subsequently } \tau_1^c = \{X, \emptyset, \{7,9\}, \{7,8\}, \{7,8,9\}, \{7\}\}, \tau_2^c = \{X, \emptyset, \{6,7\}, \{8\}, \{6,7,8\}, \{7\}\}$$

$$\text{Subsequently pre-open}(\tau_1) = \{\{6\}, \{6,7\}, \{6,8\}, \{6,9\}, \{6,7,9\}, \{6,7,8\}, \{6,8,9\}, X, \emptyset\}$$

Now  $\{6,7\}, \{6,7,9\}, \{6,7,8\}$  are not top.-related but rather  $per$ -open  $\tau_1$

$$\text{Furthermore pre-open}(\tau_2) = \{\{9\}, \{6,9\}, \{7,9\}, \{8,9\}, \{6,7,9\}, \{6,8,9\}, \{7,8,9\}, X, \emptyset\}$$

Now  $\{6,9\}, \{7,9\}, \{6,7,8\}, \{7,8,9\}$  are belong to  $per$ -open but not belong to top. ( $\tau_2$ )

**2.11.Note:** Given a family of top. spaces on  $X$ , denoted as  $\{\tau_k\}_{k \in J}$ ,  $\mathcal{T} \in \tau_k$  for every  $k \in J$ . If  $\mathcal{T}$  is  $pr$ -open, it need not necessarily be part of the intersection of top..

**2.12. Example:** take  $X = \{6,7,8,9\}$

$$\tau_1 = \{X, \emptyset, \{8,9\}, \{9\}, \{7,9\}, \{7,8,9\}\}, \tau_2 = \{X, \emptyset, \{8,9\}, \{7,9\}, \{9\}, \{7,8,9\}\}$$

$$\tau_3 = \{X, \emptyset, \{8\}, \{9\}, \{6,8,9\}, \{8,9\}\}$$

$$\bigcap_{k=1}^3 \tau_k = \{X, \emptyset, \{9\}, \{8,9\}\}$$

$$\left( \bigcap_{k=1}^3 \tau_k \right)^c = \{X, \emptyset, \{6,7,8\}, \{6,7\}\}$$

We obtain and own the power set. Next

$$\text{pre-open}(\tau_1) = \{\{8\}, \{9\}, \{9\}, \{8,9\}, \{6,8,9\}, \{7,8,9\}, X, \emptyset\}$$

Now  $\{8\}, \{7,9\}, \{6,8,9\}$  and  $\{7,8,9\}$  are belong to  $\text{per}\{\tau_1\}$  but not belong to  $\bigcap_{k=1}^3 \tau_k$ .

$$\text{Also pre-open}(\tau_2) = \{\{9\}, \{6,9\}, \{7,9\}, \{8,9\}, \{6,7,9\}, \{6,8,9\}, \{7,8,9\}, X, \emptyset\}$$

Now  $\{6,9\}, \{7,9\}, \{6,7,8\}, \{6,7,9\}$  and  $\{7,8,9\}$  are included in  $\text{per}\{\tau_2\}$  but excluded from  $\bigcap_{k=1}^3 \tau_k$

$$\text{pre-open}(\tau_3) = \{\{9\}, \{8\}, \{8,9\}, \{6,8,9\}, \{7,8,9\}, X, \emptyset\}$$

Now  $\{8\}, \{6,8,9\}$  and  $\{7,8,9\}$  are belong to  $\text{per}\{\tau_3\}$  but not belong to  $\bigcap_{k=1}^3 \tau_k$ .

**2.13.Note:** For every  $k$  in  $X$ , let  $\{\tau_k\}_{k \in J}$  be a family of top. spaces. If  $\mathcal{T}$  is  $pr$ -open, it need not necessarily belong to  $\nu$ -open, and vice versa.

**2.14. Example:** take  $X = \{6,7,8\}$

$$\tau_1 = \{X, \emptyset, \{6\}, \{7\}, \{7,8\}\}, \tau_2 = \{X, \emptyset, \{6,7\}, \{6,8\}, \{6\}, \{8\}\}$$

$$\bigcap_{k=1}^2 \tau_k = \{X, \emptyset, \{6\}, \{6,7\}\}$$

$\nu$ -open set =  $\{X, \emptyset, \{6,7\}, \{6,8\}, \{6\}, \{8\}\}$ . We get

$$\text{pre-open set } (\tau_1) = \{\{6\}, \{7\}, \{6,7\}, \{7,8\}, X, \emptyset\}$$

Now  $\{7\}$  and  $\{7,8\}$  are belong to  $\text{per}\{\tau_1\}$  open set but are outside of the  $\nu$ -open set.

Also  $\{8\}$  belong to  $\text{per}\{\tau_2\}$  open but are outside of the  $\nu$ -open set.

**2.15.Note:** Let  $\{\tau_k\}_{k \in J}$  be a family of top. space on  $X$ ,  $\mathcal{T} \in \tau_k$  for all  $k \in J$ , if  $\mathcal{T} \in \nu$ -open, Subsequently not necessary belong to pre-open.

For every  $k$  in  $J$ , let  $\{\tau_k\}_{k \in J}$  be a family of top. spaces on  $X$  such that  $\mathcal{T} \in \tau_k$ ; if  $\mathcal{T}$  is  $\nu$ -open, it need not belong to  $pr$ -open.

**2.16. Example:** take  $X = \{6,7,8\}$

$$\tau_1 = \{X, \emptyset, \{8\}, \{7\}, \{7,8\}\}, \tau_2 = \{X, \emptyset, \{6,7\}, \{7,8\}, \{6\}, \{7\}\}$$

$$\bigcap_{k=1}^2 \tau_k = \{X, \emptyset, \{7\}, \{7,8\}\}$$

$\nu$ -open set =  $\{X, \emptyset, \{6,7\}, \{7,8\}, \{7\}\}$ ,  $\tau_1$  complement =  $\{X, \emptyset, \{1\}, \{1,3\}, \{2,1\}\}$

pre-open set  $(\tau_1) = \{\{8\}, \{7\}, \{7,8\}, X, \emptyset\}$ . Now  $\{6,7\}$  belong to  $\nu$ -open set but not belong to  $\text{pre}\{\tau_1\}$  open set. Also  $\{8\}$  belong to  $\text{pre}\{\tau_2\}$  open set but are outside of the  $\nu$ -open set.

**2.17.Note:**  $\mathcal{T}$  is  $pr_\nu$ -open if and only if  $\{\tau_k\}_{k \in J}$  is a family of top. space on  $X$  and  $\mathcal{T} \in \tau_k$  for every  $k \in J$ .

Proof : Consider an arbitrary open set  $\mathcal{A}$

Subsequently  $\mathcal{A} = \text{int}(\mathcal{A})$ . Considering  $\mathcal{A} \subseteq \text{cl}(\mathcal{A}) \Rightarrow \mathcal{A} =$

$\text{int}(\mathcal{A}) \subseteq \text{int}(\text{cl}(\mathcal{A}))$

$\mathcal{A} \subseteq \text{int}(\text{cl}(\mathcal{A}))$

**2.18.Note:** Take  $\{\tau_k\}_{k \in J}$  be a family of top. spaces on  $X$ , such that  $\mathcal{T} \in \tau_k$  for every  $k \in J$ . If  $\mathcal{T}$  is not necessarily a member of  $\tau$  if it is inside  $pr_{\nu}$ -open.

**2.19. Example:** Take  $X = \{6,7,8\}$

$$\tau_1 = \{X, \emptyset, \{6,8\}, \{8\}, \{6\}, \{7,8\}\}, \tau_2 = \{X, \emptyset, \{6,8\}, \{6\}, \{6,7\}, \{7\}\}$$

$$\bigcap_{k=1}^2 \tau_k = \{X, \emptyset, \{6,8\}, \{6\}\}$$

$$\nu\text{-open set} = \{X, \emptyset, \{6,8\}, \{6\}, \{6,7\}\}, \nu^c\text{-open set} = \{X, \emptyset, \{7\}, \{7,8\}, \{8\}\}$$

$$\text{Subsequently } pr_{\nu}\text{-open set}(\tau_1) = \{\{6\}, \{6,7\}, \{6,8\}, X, \emptyset\}$$

$$\tau_1^c = \{\{7\}, \{7,8\}, \{6,8\}, X, \emptyset\}. \quad \text{Subsequently pre-open set}(\tau_1) = \{\{6\}, \{8\}, \{6,8\}, \{7,8\}, X, \emptyset\}$$

Now  $\{1,2\}$  appropriate to  $pre\text{-}\nu(\tau_1)$  but not be appropriate to  $pre(\tau_1)$ .

**2.20.Note:** Let  $\tau_k$  for all  $k \in J$  is family of top. space on  $X$ ,  $\mathcal{T} \in \tau_k$  for all  $k \in J$ , if  $\mathcal{T} \in pre\text{-open}$ , therefore not required appropriate to  $pr_{\nu}$ -open.

**2.21. Example:** Let  $X = \{6,7,8\}$

$$\tau_1 = \{X, \emptyset, \{6\}, \{7\}, \{6,7\}, \{7,8\}\}, \tau_2 = \{X, \emptyset, \{7,8\}, \{6,8\}, \{8\}, \{7\}\}$$

$$\bigcap_{k=1}^2 \tau_k = \{X, \emptyset, \{7\}, \{7,8\}\}$$

$$\nu\text{-open set} = \{X, \emptyset, \{6,7\}, \{7,8\}, \{7\}\}, \tau_1^c = \{X, \emptyset, \{7,8\}, \{6,8\}, \{8\}, \{6\}\}$$

$$pr_{\nu}\text{-top.}(\tau_1) = \{\{6,7\}, \{7\}, \{7,8\}, X, \emptyset\}$$

$$pre\text{-top.}(\tau_1) = \{\{6\}, \{7\}, \{6,7\}, \{7,8\}, X, \emptyset\}$$

$\{6\} \in pre\text{-top.}(\tau_1)$  but not appropriate to  $pr_{\nu}\text{-top.}(\tau_1)$ .

$$\text{Also } \tau_2^c = \{X, \emptyset, \{6,8\}, \{6\}, \{7\}, \{6,7\}\}$$

$pre\text{-top.}(\tau_2) = \{\{8\}, \{7\}, \{6,8\}, \{7,8\}, X, \emptyset\}$ . Now  $\{6,8\} \in pre\text{-top.}(\tau_2)$  but not appropriate to  $pr_{\nu}\text{-top.}(\tau_2)$ .

### 3- Internal and External Comparison between pre- $\nu$

**3.1.Theorem:** take  $(X, pre_{\nu o X})$  is a  $pr_{\nu}$ -space on  $X$ , when  $\mathcal{A}$  subset of  $X$ .

Subsequently  $int_{pre_{\nu}}(\mathcal{A}) = \text{union of all } i \in I \text{ for } \{\mathbb{X}_i: \mathbb{X}_i \text{ is } pr_{\nu}\text{-open, } \mathbb{X}_i \subseteq \mathcal{A}, \forall i \in I\}$

Proof: take  $k \in int_{pre_{\nu}}(\mathcal{A})$ , Subsequently  $k$  is an  $pr_{\nu}$ -interior point of  $\mathcal{A}$ , therefore Here is  $\mathbb{X} \in pr_{\nu o X}$ . in in a manner that  $k \in \mathbb{X} \subseteq \mathcal{A}$ . Now  $\mathbb{X}$  is  $pre_{\nu}$ -open in a manner that  $\mathbb{X} \subseteq \mathcal{A}$  Thus, we're left with  $k \in \text{union of all } i \in I \text{ for } \{\mathbb{X}_i: \mathbb{X}_i \text{ is } pr_{\nu}\text{-open, } \mathbb{X}_i \subseteq \mathcal{A}, \forall i \in I\}$  that's is suggests that,  $int_{pr_{\nu}}(\mathcal{A}) \subseteq \text{union of all } i \in I \text{ for } \{\mathbb{X}_i: \mathbb{X}_i \text{ is } pr_{\nu}\text{-open, } \mathbb{X}_i \subseteq \mathcal{A}, \forall i \in I\}$ .

Next take  $k \in \bigcup_{i \in I} \{\mathbb{X}_i: \mathbb{X}_i \text{ is } pr_{\nu}\text{-open, } \mathbb{X}_i \subseteq \mathcal{A}, \forall i \in I\}$

Subsequently Here is  $pr_{\nu}$ -open  $\mathbb{X}^*$ ,  $\mathbb{X}^* \subseteq \mathcal{A}$  in a manner that

$k \in \mathbb{X}^*$  therefore  $k \in \mathcal{A}$  is an  $pr_{\nu}$ -interior point of  $\mathcal{A}$ , thus

$$int_{pr_{\nu}}(\mathcal{A}) \supseteq \bigcup_{i \in I} \{\mathbb{X}_i: \mathbb{X}_i \text{ is } pr_{\nu}\text{-open, } \mathbb{X}_i \subseteq \mathcal{A}, \forall i \in I\}$$

**3.2.Theorem:** Take  $(X, pr_{\nu o X})$  be a  $pr_{\nu}$ -space and Assume that  $\mathcal{A}, \mathcal{B} \subset X$ .

$$int_{pre_{\nu}}(\emptyset) = \emptyset \text{ and } int_{pre_{\nu}}(X) = X.$$

$$\text{If } \mathcal{A} \text{ subset } \mathcal{B} \Rightarrow int_{pre_{\nu}}(\mathcal{A}) \subseteq int_{pre_{\nu}}(\mathcal{B})$$

Proof:

1. Considering  $\emptyset$ , and  $X$  are  $pr_{\nu}$ -open sets, Subsequently the following is the outcome.
2. Assume  $\mathcal{A} \subseteq \mathcal{B}$ , considering  $\text{int}_{pr_{\nu}}(\mathcal{A})$  is a  $pr_{\nu}$ -open set included in  $\mathcal{A}$ , and  $\mathcal{A} \subseteq \mathcal{B}$  Subsequently  $\text{int}_{pr_{\nu}}(\mathcal{A})$  is a  $pr_{\nu}$ -open set included in  $\mathcal{B}$ . However,  $\text{int}_{pr_{\nu}}(\mathcal{B})$  is the largest  $pr_{\nu}$ -open set included in  $\mathcal{B}$ , that's suggests to  $\text{int}_{pr_{\nu}}(\mathcal{A}) \subseteq \text{int}_{pr_{\nu}}(\mathcal{B})$ .

**3.3.Proposition:** Take  $\tau_k$  when  $k \in J$  is a family of top. space on  $X$ , and  $\mathcal{A} \in \bigcup_{k \in J} \tau_k$  Subsequently  $\text{int}(\mathcal{A})$  in  $pr_{\nu} = \text{int}(\mathcal{A})$  in  $\beta$ - $\nu$ -open but the  $\text{int}(\mathcal{A})$  in  $\beta$ eta( $\nu \circ X$ ), and  $\text{pre}(\nu \circ X)$  is not equal to  $\text{int}(\mathcal{A})$  in  $\nu \circ X$ .

**3.4.Example:** Let  $X = \{6,7,8\}$

$$\tau_1 = \{X, \emptyset, \{8\}, \{6,8\}, \{7,8\}\}, \tau_2 = \{X, \emptyset, \{6,8\}, \{6,7\}, \{6\}\}.$$

$$\bigcap_{k=1}^2 \tau_k = \{X, \emptyset, \{6,8\}\}$$

$$\nu\text{-open set} = \{X, \emptyset, \{6,8\}\}, \nu^c\text{-open set} = \{X, \emptyset, \{7\}\}$$

$$\text{We consider the power points} = \{X, \emptyset, \{6\}, \{7\}, \{8\}, \{6,7\}, \{6,8\}, \{7,8\}\}$$

$$pr_{\nu}\text{-open set} = \{X, \emptyset, \{6\}, \{8\}, \{6,7\}, \{6,8\}, \{7,8\}\}$$

$$\beta\text{-}\nu\text{-open set} = \{X, \emptyset, \{6\}, \{8\}, \{6,7\}, \{6,8\}, \{7,8\}\}$$

Now  $\{6,8\}$  belong to  $\bigcap_{k=1}^2 \tau_k$  and belong to  $\nu$ -open and belong to  $pr_{\nu}$ . And  $\text{int}(\{6,8\}) = \{6,8\}$  in  $(X, \tau_k)$ . And  $\text{int}(\{6,8\}) = \{6,8\}$  in  $\nu$ -open set. And  $\text{int}(\{6,8\}) = \{6,8\}$  in  $\beta$ - $\nu$ -open

but  $\text{int}_{\nu}(\{6\}) = \{6\}$  in  $pr_{\nu}$ -open,  $\text{int}_{\nu}(\{6\}) = \emptyset$  in  $(X, \nu \circ X)$ , and  $\text{int}_{\nu}(\{6\}) = \{6\}$  in  $\beta$ - $\nu$ -open set. Show we conclude that  $\text{int}(\{6\})$  in  $pr_{\nu}$ -open set  $\neq$   $\text{int}(\{6\})$  in  $(X, \nu \circ X)$ .

**3.5.Proposition:** take  $\{\tau_k\}_{k \in J}$  be a family of top. space on  $X$ , and  $\mathcal{T} \in \tau_k$  for  $k \in J$ , Subsequently  $\text{cl}(\mathcal{A}) = \text{cl}_{\nu}(\mathcal{A})$  in  $(X, \tau_k)$  for all  $k \in J$ .

**3.6.Theorem:** Assume that  $\mathcal{A} \subseteq X$  and that  $(X, pr_{\nu \circ X})$  is a  $pr_{\nu}$ -space on  $X$ . Consequently, the smallest  $\nu$ -closed set that contains  $\mathcal{A}$  is  $\text{cl}_{pr_{\nu}}(\mathcal{A})$ .

Proof: We see that the intersection of  $pr_{\nu}$ -closed sets is  $pr_{\nu}$ -closed set, hence we've  $\text{cl}_{pr_{\nu}}(\mathcal{A})$  is a  $pr_{\nu}$ -closed set  $\mathcal{A} \subseteq \text{cl}_{pr_{\nu}}(\mathcal{A})$ . Based on the given definition  $\text{cl}_{pr_{\nu}}(\mathcal{A})$ . Let  $\mathcal{A}^*$  be a  $pr_{\nu}$ -closed set that include  $\mathcal{A}$ . Subsequently  $\mathcal{A}^*$  incorporate the intersection of all  $pr_{\nu}$ -closed set that include  $\mathcal{A}$ . Therefore  $\text{cl}_{pr_{\nu}}(\mathcal{A}) \subseteq \mathcal{A}^*$

**3.7.Proposition:** Let  $\{\tau_k\}_{k \in J}$  be a family of top. space on  $X$ , and  $\mathcal{T} \in \tau_k$  for  $k \in J$ , Subsequently  $\text{cl}(\mathcal{A})$  in  $(X, \bigcap_k \tau_k) = \text{cl}_{\nu}(\mathcal{A})$  in  $(X, \nu \circ X) = \text{cl}_{\nu}(\mathcal{A})$   $pr_{\nu}$ -open but  $\text{cl}_{\nu}(\mathcal{A})$  of  $pr_{\nu}$ -open is not comparable to  $\text{cl}_{\nu}(\mathcal{A})$  in  $(X, \nu \circ X)$ .

**3.8.Example:** Let  $X = \{6,7,8\}$

$$\tau_1 = \{X, \emptyset, \{6\}, \{6,7\}, \{7\}\}, \tau_2 = \{X, \emptyset, \{6,7\}\}$$

Now

$$\bigcap_{k=1}^2 \tau_k = \{X, \emptyset, \{6,7\}\}$$

$$\text{And } \nu\text{-0open set} = \{X, \emptyset, \{6\}, \{6,7\}, \{7\}, \{6,8\}, \{7,8\}\}$$

Now  $\{6,7\}$  belong to  $\bigcap_{k=1}^2 \tau_k$  and belong to  $\nu$ -open set and belong to  $pr_{\nu}$ -open set  $\text{cl}\{6,7\} = X$  in  $(X, \tau_k)$ ,  $(X, \nu \circ X)$ , and  $pr_{\nu}$ -open set but  $\text{cl}_{\nu}\{6\} = \{6\}$  in  $pr_{\nu}$ -open set,

and  $\text{cl}\{6\} = X$  in  $\nu$ -open set.

So we have  $\text{cl}\{6\}$  in  $pr_\nu$ -open set  $\neq \text{cl}\{6\}$  in  $(X, \nu \circ X)$ .

**3.9.Definition:** Take  $(X, \nu \circ X)$  is a  $\nu$ -space on  $X$ , and  $\mathcal{T} \subseteq X$ . A point  $b \in X$  is contacted an  $\nu$ -exterior point of  $A$ , if any  $N \in \nu \circ X$  such a way that  $b \in N \subseteq A^c$ .

**3.10.Definition:** The collection of all  $\nu$ -exterior points of  $A$  referred to as the  $\nu$ -exterior of  $A$  and indicated by  $\text{ext}_\nu(A)$ .

**3.11.Proposition:** Let  $\{\tau_k\}_{k \in J}$  be a family of top. space on  $X$ , and  $\mathcal{T} \in \tau_k$  for  $k \in J$ , Subsequently  $\text{ext}_\nu(A)$  in  $(X, \bigcap_k^2 \tau_k) = \text{ext}_\nu(A)$  in  $(X, \nu \circ X) = \text{ext}_\nu(A)$  in  $pr_\nu$ -open but  $\text{ext}_\nu(A)$  in  $pr_\nu$ -open  $\neq \text{ext}_\nu(A)$  in  $(X, \nu \circ X)$ .

**3.12.Theorem:** Assume  $(X, pr_{\nu \circ X})$  be a  $pr_\nu$ -space on  $X$ , and  $\mathcal{A}$  subset  $X$ . Subsequently  $\text{ext}_{pr_\nu}(\mathcal{A})$  equal to  $\text{int}_{pr_\nu}(\mathcal{A}^c)$ .

Proof: Let's assume that  $b$  is an element of the set of points exterior to  $\mathcal{A}$ , denoted as  $\text{ext}_{pr_\nu}(\mathcal{A})$ . Consequently,  $b$  is an exterior point of  $\mathcal{A}$  with respect to  $pr_\nu$ . This means that there exists a set  $M$ , which is an open set with respect to  $pr_\nu$ , such that  $b$  is an interior point of  $M$  and  $M$  is a subset of the complement of  $\mathcal{A}$ , denoted as  $\mathcal{A}^c$ . Therefore,  $b$  is also an interior point of the complement of  $\mathcal{A}$  with respect to  $pr_\nu$ , denoted as  $\text{int}_{pr_\nu}(\mathcal{A}^c)$ . Hence, we can conclude that  $\text{ext}_{pr_\nu}(\mathcal{A})$  is a subset of  $\text{int}_{pr_\nu}(\mathcal{A}^c)$ . Let's assume that  $b$  belongs to the interior of the complement of  $\mathcal{A}$ , denoted as  $\text{int}_{pr_\nu}(\mathcal{A}^c)$ . Consequently,  $b$  is an interior point of  $\mathcal{A}$  with respect to  $pr_\nu$ . This means that there exists a set  $M$ , which belongs to the  $pr_\nu$ -exterior of  $X$ , such that  $b$  belongs to  $M$  and  $M$  is a subset of the complement of  $\mathcal{A}$ . This follows from the definition of  $pr_\nu$ -exterior. Given that  $b$  is a proper exterior point of  $\mathcal{A}$ , we can conclude that  $b$  belongs to the set of proper exterior points of  $\mathcal{A}$ .

**3.13.Example:** Let  $X = \{6,7,8,9\}$

$$\tau_1 = \{X, \emptyset, \{7,8\}, \{8,9\}, \{8\}\}, \tau_2 = \{X, \emptyset, \{6,8,9\}, \{8,9\}, \{6\}\}.$$

Now

$$\bigcap_{k=1}^2 \tau_k = \{X, \emptyset, \{8,9\}\}$$

$$\nu\text{-open} = \{X, \emptyset, \{8,9\}, \{6,8,9\}, \{7,8,9\}\}, \quad \nu\text{-complement} = \{X, \emptyset, \{6,7\}, \{7\}, \{6\}\}$$

$$pr_\nu\text{-open} = \left\{ \begin{array}{l} X, \emptyset, \{8\}, \{9\}, \{6,8\}, \{6,9\}, \{7,8\}, \{7,9\}, \{8,9\}, \{6,7,8\} \\ \{6,7,9\}, \{6,8,9\}, \{7,8,9\} \end{array} \right\}$$

$\{6,7\}$  belong to  $\bigcap_{k=1}^2 \tau_k$ ,  $\nu$ -open and  $pr_\nu$ -open

$\text{ext}_\nu(\{8,9\}) = \{6,7\}^c = \emptyset$  in  $(X, \bigcap_{k=1}^2 \tau_k)$ ,  $(X, \nu \circ X) = \text{ext}_\nu(A)$  in  $pr_\nu$ -open, but  $\text{ext}_\nu(\{6,7,8\}) = \{9\}^c = 9$ ,  $\{6,7,8\}$  belong to  $pr_\nu$ -open, but  $\text{ext}_\nu(\{6,7,8\}) = \emptyset$  in  $\nu$ -open.

**3.14.Note:** Let  $\{\tau_k\}_{k \in J}$  be a family of top. space on  $X$ , and  $A \in \bigcap_k^2 \tau_k$  for  $k \in J$ , thus  $b(A)$  in  $(X, \bigcap_k^2 \tau_k) = b(A)$  in  $(X, \nu o X) = b(A)$  in  $pr_{\nu}$ -open but  $b(A)$  not equal between  $pr_{\nu}$ -open and in  $\bigcap_k^2 \tau_k$ .

**3.15.Example:** Let  $X = \{6,7,8\}$

$$\tau_1 = \{X, \emptyset, \{6\}, \{7\}, \{6,7\}\}, \tau_2 = \{X, \emptyset, \{7\}, \{6,7\}, \{8\}, \{7,8\}\}$$

Now

$$\bigcap_{k=1}^2 \tau_k = \{X, \emptyset, \{7\}, \{6,7\}\}$$

$$\text{And } \nu\text{-open set} = \{X, \emptyset, \{7\}, \{6,7\}, \{7,8\}\},$$

$$pr_{\nu}\text{-open set} = \{X, \emptyset, \{7\}, \{6,7\}, \{7,8\}\}$$

The open sets  $\{6\}$  and  $\{6,7\}$  belong to  $\bigcap_{k=1}^2 \tau_k$ ,  $\nu$ -open and pre- $\beta$ - $\nu$ -open set

$$b(\{7\}) = \{6,8\} \text{ in } (X, \bigcap_{k=1}^2 \tau_k), (\{7\}) = \{6,8\} \text{ in } \nu\text{-open set}$$

$$b(\{6,7\}) = \{8\} \text{ in } (X, \bigcap_{k=1}^2 \tau_k), (\{6,7\}) = \{8\} \text{ in } \nu\text{-open set}$$

$$b(\{7\}) = \{6,8\} \text{ in } (X, \bigcap_{k=1}^2 \tau_k), (\{7\}) = \{6,8\} \text{ in } pr_{\nu}\text{-open set,}$$

Subsequently but  $b(A)$  in  $(X, \bigcap_{k=1}^2 \tau_k) = b(A)$  in  $(X, \nu o X) = b(A)$  in  $pr_{\nu}$ -open set  $\{7,8\} \in \nu$ -open set, but  $\{7,8\}$  not belong to  $(X, \bigcap_{k=1}^2 \tau_k)$

$$b(\{7,8\}) = \{6,7,8\}/\{7,8\} \cup \emptyset = \{6\} \text{ in } \nu\text{-open set, but } b(\{7,8\}) = \{6,7,8\}/\{7\} \cup \emptyset = \{6,8\} \text{ in } (X, \bigcap_{k=1}^2 \tau_k), b(A) \text{ in } \nu\text{-open set} \neq b(A) \text{ in } (X, \bigcap_{k=1}^2 \tau_k) \text{ when } A \notin \nu\text{-open set}$$

**3.16.Note:** Boundary to open set  $A$   $b(A)$  in  $pr_{\nu}$ -open is not equal to  $b(A)$  in  $\nu$ -open.

**3.17.Example:** Let  $X = \{6,7,8\}$

$\tau_1 = \{X, \emptyset, \{8\}, \{6,8\}\}, \tau_2 = \{X, \emptyset, \{6\}, \{6,8\}\}$ . Subsequently  $(X, \tau_1), (X, \tau_2)$  are top. spaces. Now

$$\bigcap_{k=1}^2 \tau_k = \{X, \emptyset, \{6,8\}\}$$

$$\text{And } \nu\text{-open} = \{X, \emptyset, \{6,8\}\} pr_{\nu}\text{-open} = \{X, \emptyset, \{6\}, \{8\}, \{6,8\}, \{7,8\}\}$$

The open  $\{7,8\}$  belong to  $\nu$ -open, but not belong to  $\nu$ -open

$$b(\{7,8\}) = \{6,7,8\}/\{7,8\} \cup \{6\} = \emptyset, b(\{7,8\}) = \{6,7,8\}/\emptyset \cup \emptyset = \{6,7,8\} = X$$

$b(A)$  in  $pr_{\nu}$ -open  $\neq b(A)$  in  $\nu$ -open.

**3.18.Note:** Let  $\{\tau_k\}_{k \in J}$  be a family of top. space on  $X$ , and  $A \in \bigcap_k^2 \tau_k$  for  $k \in J$ , Subsequently  $D(A)$  in  $(X, \bigcap_k^2 \tau_k) = D(A)$  in  $(X, \nu o X) = D(A)$  not equal to in  $pr_{\nu}$ -open

**3.20.Example:** Let  $X = \{6,7,8\}$

$$\tau_1 = \{X, \emptyset, \{6\}, \{6,7\}, 7\}, \tau_2 = \{X, \emptyset, \{6,7\}\}. \text{ Now}$$

$$\bigcap_{k=1}^2 \tau_k = \{X, \emptyset, \{6,7\}\}$$

$$\text{And } \nu\text{-open} = \{X, \emptyset, \{6,7\}\}, pr_{\nu}\text{-open} = \{X, \emptyset, \{6\}, \{7\}, \{6,7\}, \{6,8\}, \{7,8\}\}$$

The open  $\{6,7\}$  belong to  $(X, \bigcap_k^2 \tau_k), (X, \nu o X)$ , and  $pr_{\nu}$ -open

$$D(\{6,7\}) = X \text{ in } (X, \bigcap_k^2 \tau_k), \text{ and } (X, \nu o X), \text{ but } D(\{6,7\}) = \{8\} \text{ in } (X, pr_{\nu o X}).$$

**3.21.Proposition:** Let  $\{\tau_k\}_{k \in J}$  be a family of top. space on  $X$ , and  $A \in \bigcap_k^2 \tau_k$  for  $k \in J$ , Subsequently  $D(A)$  in  $pr_{\nu}$ -open not equal to  $D(A)$  in  $\nu$ -open.

### 3.22.Example: Let $X = \{6,7,8\}$

$$\tau_1 = \{X, \emptyset, \{7,8\}, \{7\}\}, \tau_2 = \{X, \emptyset, \{7,8\}, \{8\}, \{6\}, \{6,8\}\}.$$

$$\bigcap_{k=1}^2 \tau_k = \{X, \emptyset, \{7,8\}\}$$

And  $\nu$ -open set=  $\{X, \emptyset, \{7,8\}\}$ , pre- $\nu$ -open set=  $\{X, \emptyset, \{7\}, \{8\}, \{6,7\}, \{6,8\}, \{7,8\}\}$

$\beta$ - $\nu$ -open set=  $\{X, \emptyset, \{7\}, \{8\}, \{6,7\}, \{6,8\}, \{7,8\}\}$

The open set  $\{7,8\}$  belong to  $(X, \bigcap_k^2 \tau_k)$ ,  $(X, \nu o X)$ , and  $pr_{\nu}$ -open set

$D(\{7,8\}) = X$  in  $(X, \bigcap_{k=1}^2 \tau_k)$ ,  $D(\{2,3\}) = X$  in  $(X, \nu o X)$

Subsequently  $D(\{7,8\}) = X$  in  $(X, \bigcap_k^2 \tau_k) = D(\{7,8\})$  in  $(X, \nu o X)$ . But  $D(\{7,8\}) = \{6\}$  in  $pr_{\nu}$ -open set.  $D(\{6,7\}) = \emptyset$  in  $pr_{\nu}$ -open set. But  $D(\{6,7\}) = \{6,8\}$  in  $\nu$ -open set. Subsequently  $D(A)$  in pre- $\nu$ -open set  $\neq D(A)$  in  $\nu$ -open set.

### Conclusion:

- If  $\{\tau_k\}_{k \in J}$  be a family of topological space on  $X$ , and  $\mathcal{T} \in \tau_k$  for  $k \in J$ , if  $\mathcal{T} \in pre\text{-open}$ , then not necessary belong to  $pr_{\nu}$ -open set also the converse is not true.
- Also every  $\nu$ -open set is  $pr_{\nu}$ -open set but the converse is not true.
- Every  $pr_{\nu}$ -open set is  $\beta_{\nu}$ -open set.
- if  $\{\tau_k\}_{k \in J}$  be a family of topological space on  $X$ , and  $\mathcal{T} \in \tau_k$  for  $k \in J$ , if  $\mathcal{T} \in pr_{\nu}$ -open set  $pr_{\nu}$ -open set, then not necessary belong to topology, also the converse is not true.

### Acknowledgment

I would you thank my supervisor Assistant Professor Lama Saad Abdel-Baqi for suggesting the topic of this work, and for advice, at least I would like also to thank my family, and my colleagues.

### References

- [1] Sayed, O. R. "Supra pre-open sets and supra pre-continuity on topological spaces." *Scientific Studies and Research* 20.2 (2010).
- [2] Mooakher, Ali Habibi, Mohammad Ebrahimi, and Amin Talabeigi. "Extraction of Pre-open Sets as a Generalized Topology from Topological Spaces." *Iranian Journal of Science and Technology, Transactions A: Science* 46.1 (2022): 203-210.
- [3] Jun, Young Bae, et al. "Applications of pre-open sets." *Applied general topology* 9.2 (2008): 213-228.
- [4] Tapi, U. D., Ranu Sharma, and Bhagyashri A. Deole. "Semi-open sets and Pre open sets In Tri Topological space." *i-Manager's Journal on Mathematics* 5.3 (2016): 41.
- [5] Rao, V. Venkateswara, and Y. Srinivasa Rao. *Neutrosophic Pre-open sets and Pre-closed sets in Neutrosophic Topology*. Infinite Study, 2017.
- [6] Sathishmohan, P., V. Rajendran, and P. K. Dhanasekaran. "Further properties of nano pre- $T_0$ , nano pre- $T_1$  and nano pre- $T_2$  spaces." *Malaya Journal of Matematik* 7.01 (2019): 34-38.
- [7] Sameer, Zaman T., and Luma S. Abdalbaqi. " $\nu$ -Open set and some of its properties." *AIP Conference Proceedings*. Vol. 2845. No. 1. AIP Publishing, 2023.
- [8] Mashhour, A. S., ME Abd El-Monsef, and I. A. Hasanein. "On pretopological spaces." *Bulletin Mathematique de la Societe des Sciences Mathematiques de la Republique Socialiste de roumanie* (1984): 39-45.
- [9] Sierpinski, Waclaw. *General topology*. Courier Dover Publications, 2020.

بعض الخصائص على المجاميع المفتوحة من النمط  $vpre$ -

انعام عويد خضير\*، لمى سعد عبدالباقي  
قسم الرياضيات، كلية التربية للبنات، جامعة تكريت، العراق  
البحث مستل من رسالة ماجستير الباحث الاول

## الخلاصة:

دراسة هذا البحث تنال تعريف  $openpre_v$ - من خلال دراسة سابقة لفضاء  $v$ - $open$  ودراسة النقاط الداخلية والخارجية. فتناولنا في هذه الورقة البحثية العلاقة بين  $openpre_v$  و  $open\beta_v$  و  $openpre$  وبين  $openpre_v$  وكذلك تمت اجراء مقارنة النقاط الداخلية والخارجية والانغلاق والنقاط الحدودية والغاية مع الامثلة والمبرهنات لـ  $openpre_v$ .

## معلومات البحث:

تأريخ الاستلام: 2023/07/10

تاريخ التعديل : 2023/07/31

تأريخ القبول: 2023/08/04

تاريخ النشر: 2024/12/30

## الكلمات المفتاحية:

$-open, pre-voX, pre$   
 $pre c(voX), per o(voX),$   
 $pre-v-open, \beta v-open,$   
 $\beta v(voX), v-exterior, and$   
 $pre(\beta)-v-open.$

## معلومات المؤلف

الايميل:

[inam.owaid379@st.tu.edu.iq](mailto:inam.owaid379@st.tu.edu.iq)

الموبايل: 07703473484