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Strongly Pseudo Nearly_2_Absorbing Submosules

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Abstract

In this work we study a notion of strongly Pseudo Nearly 2-Absorbing submodule and some $g$eneralization, it is a generalization of 2_Absorbing submodules, and a strong form of (Nearly_2_Absorbing and Pseudo_2_A bsorbing) submodule. Characterization and examples of the suggestion a generalization are given, as well as many different suggestion properties that have been $p$roven.

Introduction:

Throughout this work, the ring R is commutative ring with identity and the module Q is unitary left $R\_$ module. A Proper submodule T of an $R\_$ module Q is called 2-Absorbing if whenever $uvh \in T$, for u, $v \in R, h \in Q$ then $uh \in T$ or $vh \in T$ or $uv \in [T :_R Q]$[1]. In 2018 introduced a notation of Nearly–2–Absorbing submodule, where a proper submodule T of Q is called Nearly_2_Absorbing submodule if whenever $uvh \in T$, for all u, $v \in R, h \in Q$, then either $uh \in T + J(Q)$ or $vh \in T + J(Q)$ or $uv\mathcal{H} \subseteq T + J(Q)$[2]. where $J(Q)$ the intersection of all maximal submodule of Q and a proper submodule $\mathcal{B}$ of an $R\_$ module Q is said to be maximal submodule if whenever $\mathcal{B} \subseteq \mathcal{L} \subseteq Q$ for some submodule $\mathcal{L}$ of Q then either $\mathcal{B} = \mathcal{L}$ or $\mathcal{L} = Q$ [3]. Newly precede a concept of Pseudo-2- Absorbing submodule, where a proper submodule T of Q is called Pseudo 2_Absorbing submodule if whenever $uvh \in T$, for all u, $v \in R, h \in Q$, then either $uh \in T + soc(Q)$ or $vh \in T + soc(Q)$ or $uv \subseteq T + soc(Q)$ [4]. Where $soc(Q)$ is the intersection of all essential submodule of Q and a non_zero submodule $\mathbb{Q}$ of Q is a essential in Q if $\mathbb{Q} \cap \mathbb{C} \neq (0)$ for any non_zero submodule $\mathbb{C}$ of Q [5]. An R_module Q is amultiplication if for any submodule T of Q is of the form $T = \mathrm{I}\mathcal{H}$ for some ideal I of R. The equivalent of $T = [T :_R Q]Q$ [6]. An R_module $\mathcal{H}$ is a faithful iff $ann_R(Q) = (0)$[7]. Finally the concept of 2-Absorbing submodule is generalized to Almost Approximaitly Nearly Prime in (2022)[8].

The Returns

Definition

A proper submodule T of an R_module Q is said to be Strongly Pseudo Nearly_2_Absorbing (for short STPN-2-A) submodule of $\mathcal{H}$, if whenever $abm \in T$, for $u, v \in R$, $h \in Q$, implies that either $uh \in T\ + (J(Q) \cap soc(Q))$ or $vh \in T\ + (J(Q) \cap soc(Q))$ or $uv \in [T\ + (J(Q) \cap soc(Q))_R : Q]$. Andideal I of a ring R is said to be STPN-2-A ideal of R if I is STPN-2-A R_submodule of an R_ module R.

Remarks and Examples

1. Let $Q = Z_{36}$, $R = Z$ and the submodule $T = \langle\bar{2}\rangle$ is STPN-2-A submodule of Q since $soc(Z_{36}) = \langle\bar{2}\rangle \cap \langle\bar{3}\rangle \cap \langle\bar{6}\rangle \cap Z_{36} = \langle\bar{6}\rangle$ and $J(Z_{36}) = \langle\bar{2}\rangle \cap \langle\bar{3}\rangle = \langle\bar{6}\rangle$ that is for all $u, v \in Z$ and $h \in Z_{36}$ such that $uvh \in \langle\bar{2}\rangle$, implies that either $uh \in T + \big(J(Z_{36}) \cap soc(Z_{36})\big) = \langle\bar{2}\rangle + (\langle\bar{6}\rangle \cap \langle\bar{6}\rangle) = \langle\bar{2}\rangle$ or $vh \in \mathcal{F} + \big(J(Z_{36}) \cap soc(Z_{36})\big) = \langle\bar{2}\rangle + (\langle\bar{6}\rangle \cap \langle\bar{6}\rangle) = \langle\bar{2}\rangle$ or $uv \in \Big[T + \big(J(Z_{36}) \cap soc(Z_{36})\big)_R : Z_{36}\Big] = 2Z$. That is $2.1.\bar{1} \in \langle\bar{2}\rangle$, impliesthat $2.\bar{1} = \bar{2} \in \langle\bar{2}\rangle$ or $2.1 = 2 \in \Big[\mathcal{F} + \big(J(Z_{36}) \cap soc(Z_{36})\big)_R : Z_{36}\Big] = 2Z$.
2. It is clear that any 2_Absorbingsubmodule of an R_ module Q is STPN-2-A but the opposite is untrue for instance:

Let $Q = Z_{36}$, $R = Z$ and the submodule $T = \langle\overline{12}\rangle$ is STPN-2-A submodule of Q since $soc(Z_{36}) = \langle\bar{2}\rangle \cap \langle\bar{3}\rangle \cap \langle\bar{6}\rangle \cap Z_{36} = \langle\bar{6}\rangle$ and $J(Z_{36}) = \langle\bar{2}\rangle \cap \langle\bar{3}\rangle = \langle\bar{6}\rangle$ that is for all $u, v \in Z$ and $h \in Z_{36}$ such that $uvh \in \langle\overline{12}\rangle$, implies that either $uh \in T + \big(J(Z_{36}) \cap soc(Z_{36})\big) = \langle\overline{12}\rangle + (\langle\bar{6}\rangle \cap \langle\bar{6}\rangle) = \langle\bar{6}\rangle$ or $vh \in T + \big(J(Z_{36}) \cap soc(Z_{36})\big) = \langle\overline{12}\rangle + (\langle\bar{6}\rangle \cap \langle\bar{6}\rangle) = \langle\bar{6}\rangle$or$uv \Big[T + \big(J(Z_{36}) \cap soc(Z_{36})\big)_R : Z_{36}\Big] = 6Z$. That is $6.2.\bar{1} \in \langle\overline{12}\rangle$, implies that $6.\bar{1} = \bar{6} \in T + \big(J(Z_{36}) \cap soc(Z_{36})\big) = \bar{6} \in \langle\overline{12}\rangle + (\langle\bar{6}\rangle \cap \langle\bar{6}\rangle) = \langle\bar{6}\rangle$ and $6.2 = 12 \in \Big[T + \big(J(Z_{36}) \cap soc(Z_{36})\big)_R : Z_{36}\Big] = 6Z$. But $T = \langle\overline{12}\rangle$ is not 2_Absorbing submodule of $\mathcal{H}$, since $2.3.\bar{2} \in \langle\overline{12}\rangle$, but $2.\bar{2} \notin \langle\overline{12}\rangle$ and $3.\bar{2} \notin \langle\overline{12}\rangle$ and $2.3 \notin [\langle\overline{12}\rangle_R : Z_{36}] = 12Z$.

3. It is visibly every STPN-2-A submodule of Q is Nearly_2_Absorbing submodule but the opposite is untrue for instance:

Let $Q = Z_{48}$, $R = Z$ and the submodule $T = \langle\overline{24}\rangle$ is Nearly_2_Absorbing submodule of Q since $soc(Z_{48}) = \langle\bar{2}\rangle \cap \langle\bar{4}\rangle \cap \langle\bar{8}\rangle = \langle\bar{8}\rangle$ and $J(Z_{48}) = \langle\bar{2}\rangle \cap \langle\bar{3}\rangle = \langle\bar{6}\rangle$ that is for all $u, v \in Z$ and $h \in Z_{48}$ such that $abm \in \langle\overline{24}\rangle$, implies that either $uh \in T + \big(J(Z_{48})\big) = \langle\overline{24}\rangle + (\langle\bar{6}\rangle) = \langle\bar{6}\rangle$ or $vh \in T + \big(J(Z_{48})\big) = \langle\overline{24}\rangle + (\langle\bar{6}\rangle) = \langle\bar{6}\rangle$. That is $2.4.\bar{3} \in \langle\overline{24}\rangle$, implies that $2.\bar{3} = \bar{6} \in \mathcal{F} + \big(J(Z_{48})\big) = \langle\overline{24}\rangle + (\langle\bar{6}\rangle) = \langle\bar{6}\rangle$ and $4.\bar{3} = \overline{12} \in T + \big(J(Z_{48})\big) = \langle\overline{24}\rangle + (\langle\bar{6}\rangle) = \langle\bar{6}\rangle$. But $T = \langle\overline{24}\rangle$ is not Nearly_2_Absorbing submodule of Q, since $2.4.\bar{3} \in \langle\overline{24}\rangle$, but $2.\bar{3} \notin \langle 24\rangle + \big(J(Z_{48}) \cap soc(Z_{48})\big) = \langle\overline{24}\rangle$ and $4.\bar{3} \notin \langle\overline{24}\rangle + \big(J(Z_{48}) \cap soc(Z_{48})\big) = \langle\overline{24}\rangle$ and $2.4 \notin \Big[\langle\overline{24}\rangle + \big(J(Z_{48}) \cap soc(Z_{48})\big)_R : Z_{48}\Big] = 24Z$.

4. It is visibly every STPN-2-A submodule of Q is pseudo_2_Absorbing submodule but the opposite is untrue for instance:

Let $Q = Z_{60}$, $R = Z$ and the submodule $T = \langle\overline{30}\rangle$ is pseudo_2_Absorbing submodule of Q since $soc(Z_{60}) = \langle\bar{2}\rangle \cap Z_{60} = \langle\bar{2}\rangle$ and $J(Z_{60}) = \langle\bar{2}\rangle \cap \langle\bar{3}\rangle \cap \langle\bar{5}\rangle = \langle\overline{30}\rangle$ that is for all $a, b \in Z$ and $m \in Z_{60}$ such that $abm \in \langle\overline{30}\rangle$, implies that either $am \in T + \big(soc(Z_{60})\big) = \langle\overline{30}\rangle + (\langle\bar{2}\rangle) = \langle\bar{2}\rangle$ or $ab \in \left[T + \big(soc(Z_{60})\big)_R : Z_{60}\right] = 2Z$. That is $2.3.\bar{5} \in \langle\overline{30}\rangle$, implies that $2.\bar{5} = \overline{10} \in T + \big(soc(Z_{60})\big) = \langle\overline{30}\rangle + (\langle\bar{2}\rangle) = \langle\bar{2}\rangle$ and $2.3 \in \left[T + \big(soc(Z_{60})\big)_R : Z_{60}\right] = 2Z$. But $T = \langle\overline{30}\rangle$ is not STPN-2-A submodule of $\mathcal{H}$, since $2.3.\bar{5} \in \langle\overline{30}\rangle$, but $2.\bar{5} \notin \langle\overline{30}\rangle + \big(J(Z_{60}) \cap soc(Z_{60})\big) = \langle\overline{30}\rangle$ and $3.\bar{5} \notin \langle\overline{30}\rangle + \big(J(Z_{60}) \cap soc(Z_{60})\big) = \langle\overline{30}\rangle$ and $2.3 \notin \left[\langle\overline{30}\rangle + \big(J(Z_{60}) \cap soc(Z_{60})\big)_R : Z_{60}\right]. = 30Z$.

5. The intersection of two STPN-2-A submodule of an $R\_$ module $\mathcal{H}$ need not be an STPN-2-A submodule . The next instance explicate:

Consider the Z_module Z_{60} and the submodule $\mathcal{L}$=$\langle\bar{5}\rangle$ and $\mathbb{Q}$=$\langle\bar{6}\rangle$ are STPN-2-A submodule of the Z_module Z_{60} (because $\langle\bar{5}\rangle$ and $\langle\bar{6}\rangle$ are 2-Absorbing of Z_{60}), but $\mathcal{L} \cap \mathbb{Q} = \langle\overline{30}\rangle$ is not STPN-2-A , (see remark and example 4).

6. If T is an STPN_2_Absorbing submodule of an $R\_$ module Q, then $[T :_R Q]$ need not to be an STPN-2-A idealof R, the next instance explicate that:

Consider the Z-module Z_{72}, the submodule $T = \langle\overline{36}\rangle$ is an STPN-2-A submodule of the Z_module Z_{72}, since $3.3.\bar{4} \in \langle\overline{36}\rangle$, implies that $3.\bar{4} \in \langle\overline{36}\rangle + \big(J(Z_{72}) \cap soc(Z_{72})\big) = \langle\overline{36}\rangle + (\langle\bar{6}\rangle \cap \langle\overline{12}\rangle) = \langle\bar{6}\rangle$, but $[\langle\overline{36}\rangle :_R Z_{72}] = 36Z$ is not to be an STPN-2-A ideal of Z, because $3.3.4 \in 36Z$,but $3.4 \notin 36Z + \big(soc(Z) \cap J(Z)\big) = 36Zand3.3 \notin \left[\langle\overline{36}\rangle + \big(J(Z) \cap soc(Z)\big)_R : Z_{60}\right] = 36Z$.

Proposition

A proper submodule T of an $R_$module Q is STPN-2-A submodule of Q if and $o$nly if forany $u, v \in R$ such that $uv \notin [T + (J(Q) \cap soc(Q))_R : Q]$ we have $[T :_Q uv] \subseteq \left[T + \big(J(Q) \cap soc(Q)\big)_Q : u\right] \cup \left[T + \big(J(Q) \cap soc(Q)\big)_Q : v\right]$.

Proof

(⇒) Suppose that T is STPN-2-A submodule of $\mathcal{H}$ and let $e \in [T :_Q uv]$, then $uve \in T$, and $uv \notin [T + (J(Q) \cap soc(Q))_R : Q]$. , it follows that either $ue \in T + (J(Q) \cap soc(Q))$ or $ve \in T + (J(Q) \cap soc(Q))$ (for T is a STPN-2-A). Thus either $e \in [T + (J(Q) \cap soc(Q))_Q : u]$ or $e \in [T + (J(Q) \cap soc(Q))_Q : v]$. Hence $e \in \left[T + \big(J(Q) \cap soc(Q)\big)_Q : u\right] \cup \left[T + \big(J(Q) \cap soc(Q)\big)_Q : v\right]$. Therefore $[T :_Q uv] \subseteq \left[T + \big(J(Q) \cap soc(Q)\big)_Q : u\right] \cup \left[T + \big(J(Q) \cap soc(Q)\big)_Q : v\right]$.

(⇐) Let $uve \in T$ for $u, v \in R$, $e \in Q$ and suppose that $uv \notin [T + (J(Q) \cap soc(Q))_R : Q]$. Then by our hypothesis $e \in [T :_Q uv \subseteq \left[T + \big(J(Q) \cap soc(Q)\big)_Q : u\right] \cup \left[T + \big(J(Q) \cap soc(Q)\big)_Q : v\right]$. It follows that $e \in \left[T + \big(J(Q) \cap soc(Q)\big)_Q : u\right] \cup \left[T + \big(J(Q) \cap soc(Q)\big)_Q : v\right]$. That is either $ue \in T + (J(Q) \cap soc(Q))$ or $ve \in T + (J(Q) \cap soc(Q))$. Therefore T is STPN-2-A submodule of Q.

Proposition

A proper submodule T of an R _ module Q is STPN-2-Asubmodule if and only if $uv\mathcal{L} \subseteq T$, for $u,v \in R$ and $\mathcal{L}$ is a submodule of Q, implies that either $u\mathcal{L} \subseteq T+(J(Q)\cap soc(Q)$ or $v\mathcal{L} \subseteq T+(J(Q)\cap soc(Q)$ or $uv \in [T+(J(Q)\cap soc(Q))_R : Q]$.

Proof

(⇒) Let T be a STPN-2-A submodule of Q and $uv\mathcal{L} \subseteq T$for $u,v \in R$, $\mathcal{L}$ is a submodule of Q. Suppose that $uv \notin [T+(J(Q)\cap soc(Q)\ _R : Q]$, $u\mathcal{L} \nsubseteq T+(J(Q)\cap soc(Q))$ and $v\mathcal{L} \nsubseteq T+(J(Q)\cap soc(Q)$. Then $\exists\ e_1, e_2 \in \mathcal{L}$ such that $ue_1 \in T+(J(Q)\cap soc(Q))$ and $ve_2 \in T+(J(Q)\cap soc(Q))$. Yet $uv \in T$ and $uv \notin [T+(J(Q)\cap soc(Q)_R : Q]$, then $e_1 \in [T :_Q uv] \subseteq [T+(J(Q)\cap soc(Q))\ _Q : u] \cup [T+(J(Q)\cap soc(Q))\ _Q : v]$, implies that $e_1 \in [T+(J(Q)\cap soc(Q))\ _Q : u] \cup [T+(J(Q)\cap soc(Q))\ _Q : v]$. But $uve_1 \in T$and $ue_1 T+(J(Q)\cap soc(Q)$, that is $e_1 \notin [T+(J(Q)\cap soc(Q)\ _Q : u]$. Thus $e_1 \in [T+(J(Q)\cap soc(Q)\ _Q : v]$, that is $ve_1 \in T+(J(Q)\cap soc(Q)$. So since $uve_1 \in T$ and $uv \notin [T+(J(Q)\cap soc(Q))\ _R : Q]$ and $ve_2 \notin T+(J(Q)\cap soc(Q)$ and $e_2 \notin [T+(J(Q)\cap soc(Q)\ _Q : v]$.Thus $e_2 \in [T+(J(Q)\cap soc(Q))_Q : u]$, it means that $ue_2 \in T+(J(Q)\cap soc(Q))$. Yet,$uv\,(e_1 + e_2) \in T$, and $uv \notin [T+(J(Q)\cap soc(Q))_R : Q]$, $uv\,(e_1 + e_2) \in T$, implies that $(e_1 + e_2) \in [T :_Q uv]$. It follows $(e_1 + e_2) \in [T+(J(Q)\cap soc(Q))_Q : u] \cup [T+(J(Q)\cap soc(Q))_Q : v]$. That is either $u(e_1 + e_2) \in T+(J(Q)\cap soc(Q))$ or $v(e_1 + e_2) \in T+(J(Q)\cap soc(Q))$. If $u(e_1 + e_2) = u\,e_1 + ue_2 \in T+(J(Q)\cap soc(Q))$ and $ue_2 \in T+(J(Q)\cap soc(Q))$, then $ue_1 \in T+(J(Q)\cap soc(Q))$ which is a contradiction. If $v(e_1 + e_2) = v\,e_1 + ve_2 \in T+(J(Q)\cap soc(Q))$ and $ve_1 \in T+(J(Q)\cap soc(Q))$, then $ve_2 \in T+(J(Q)\cap soc(Q))$ which is a contradiction. Hence either $u\mathcal{L} \subseteq T+(J(Q)\cap soc(\mathcal{H}))$ or $v\mathcal{L} \subseteq T+(J(\mathcal{H})\cap soc(\mathcal{H}))$ or $uv \in [T+(J(\mathcal{H})\cap soc(\mathcal{H}))_R : \mathcal{H}]$.

(⇐) let $uve \in T$ for $u, v \in R$, $e \in Q$, that *is* $uv(e) \subseteq T$, hence by hypothesis either $u(e) \subseteq T+(J(Q)\cap soc(Q)$ or $u(e) \subseteq T+(J(Q)\cap soc(Q))$ or $uv \in [T+(J(Q)\cap soc(Q))_R : Q]$. That is either $ue \in T+(J(Q)\cap soc(Q))$ or $ve \in T+(J(Q)\cap soc(Q))$ or $uv \in [T+(J(Q)\cap soc(Q))_R : Q]$. Hence T is STPN-2-A submodule of Q.

Proposition

A proper submodule T of an R_ module Q is a $\mathrm{STPN}-2-\mathrm{A}$ submodule of Q if and only if $IJ\mathcal{L} \subseteq T$ for some ideals I, J of R and some submodule $\mathcal{L}$ of Q, implies that either $I\mathcal{L} \subseteq T+(J(Q)\cap soc(Q))$ or $J\mathcal{L} \subseteq T+(J(Q)\cap soc(Q))$ or $IJ \subseteq [T+(J(Q)\cap soc(Q))\ _R : Q]$.

Proof

(⇒) Let if $IJ\mathcal{L} \subseteq T$ for some ideals I, J of R and some submodule $\mathcal{L}$ of Q with $IJ \nsubseteq [T+(J(Q)\cap soc(Q))_R : Q]$. To show that $I\mathcal{L} \subseteq T+(J(Q)\cap soc(Q))$ or $J\mathcal{L} \subseteq T+(J(Q)\cap soc(Q))$. Assume that $I\mathcal{L} \nsubseteq T+(J(Q)\cap soc(Q))$ and $J\mathcal{L} \nsubseteq T+(J(Q)\cap soc(Q))$, that is $\exists$ $a_1 \in I$ and $a_2 \in J$ such that $a_1\mathcal{L} \nsubseteq T+(J(Q)\cap soc(Q))$ and $a_2\mathcal{L} \nsubseteq T+(J(Q)\cap soc(Q))$. Yet, $a_1\, a_2\mathcal{L} \subseteq T$, and T is STPN-2-A submodule of Q, then $a_1\mathcal{L} \subseteq T+(J(Q)\cap soc(Q))$ or $a_2\mathcal{L} \subseteq T+(J(Q)\cap soc(Q))$) or $a_1\, a_2 \in [T+(J(Q)\cap soc(Q))\ _R : Q]$. Since $IJ \nsubseteq [T+(J(Q)\cap soc(Q))\ _R : Q]$, $\exists$ $b_1 \in I$ and $b_2 \in J$ such that $b_1\, b_2 \notin [T+(J(Q)\cap soc(Q))\ _R : Q]$. But $b_1\, b_2 L \subseteq T$ and T is STPN-2-A submodule of Q, and $b_1\, b_2 \notin [T+(J(Q)\cap soc(Q))\ _R : Q]$, then either $b_1\mathcal{L} \subseteq T+(J(Q)\cap soc(Q))$ or $b_2\mathcal{L} \subseteq T+(J(Q)\cap soc(Q))$.

State(1): If $b_1\mathcal{L} \subseteq T+(J(Q)\cap soc(Q))$ and $b_2\mathcal{L} \nsubseteq T+(J(Q)\cap soc(Q))$. Since $a_1\, b_2\mathcal{L} \subseteq T$ and $b_2\mathcal{L} \nsubseteq T+(J(Q)\cap soc(Q))$ and $a_1\mathcal{L} \nsubseteq T+(J(Q)\cap soc(Q))$, so that by proposition (2.4) $a_1\, b_2 \in [T+(J(Q)\cap soc(Q))\ _R : Q]$. Since $b_1\mathcal{L} \subseteq T+(J(Q)\cap soc(Q))$ and $a_1\mathcal{L} \nsubseteq T+(J(Q)\cap soc(Q))$ we get $(a_1+b_1)\mathcal{L} \nsubseteq T+(J(Q)\cap soc(Q))$. On the other hand $(a_1 +$

$b_1)b_2\mathcal{L} \subseteq T$ and T is STPN-2-A , and $(a_1+b_1)\mathcal{L} \nsubseteq T + (J(Q)\cap soc(Q))$, $b_2\mathcal{L} \nsubseteq T + (J(Q)\cap soc(Q))$, then by a bove proposition $(a_1+b_1)b_2 = a_1b_2 + b_1b_2 \in [T + (J(Q)\cap soc(Q))\ _R: Q]$. But $a_1\, b_2 \in [T + (J(Q)\cap soc(Q))\ _R: Q]$, it follows that $b_1\, b_2 \in [T + (J(Q)\cap soc(Q))\ _R: Q]$, which is a contradiction .

State(2): If $b_2\mathcal{L} \subseteq T + (J(Q)\cap soc(Q))$ and $b_1\mathcal{L} \nsubseteq T + (J(Q)\cap soc(Q))$, by similar steps of state (1) we get a contradiction .

State(3): If $b_1\mathcal{L} \subseteq T + (J(Q)\cap soc(Q))$ and $b_2\mathcal{L} \subseteq T + (J(Q)\cap soc(Q))$, since $b_2\mathcal{L} \subseteq T + (J(Q)\cap soc(Q))$ and $a_2\mathcal{L} \nsubseteq T + (J(Q)\cap soc(Q))$, we get $(a_2+b_2)\mathcal{L} \nsubseteq T + (J(Q)\cap soc(Q))$. But $a_1(a_2+b_2)\mathcal{L} \subseteq T$ and T is STPN-2-A with $a_1\mathcal{L} \nsubseteq T + (J(Q)\cap soc(Q))$ and $(a_2+b_2)\mathcal{L} \nsubseteq T + (J(Q)\cap soc(Q))$ we get $a_1(a_2+b_2) \in [T + (J(Q)\cap soc(Q))\ _R: Q]$. Since $a_1\, a_2 \in [T + (J(Q)\cap soc(Q))\ _R: Q]$ and $a_1a_2 + a_1b_2 \in [T + (J(Q)\cap soc(Q))\ _R: Q]$, implies that $a_1b_2 \in [T + (J(Q)\cap soc(Q))\ _R: Q]$. Yet , since $(a_1+b_1)a_2 \in T$ and $a_2\mathcal{L} \nsubseteq T + (J(Q)\cap soc(Q))$ and $(a_1+b_1)\mathcal{L} \nsubseteq T + (J(Q)\cap soc(Q))$ and T is STPN-2-A, then we get $(a_1+b_1)a_2 \in [T + (J(Q)\cap soc(Q))\ _R: Q]$. But $(a_1+b_1)a_2 = a_1a_2 + b_1a_2 \in [T + (J(Q)\cap soc(Q))\ _R: Q]$ and since $a_1\, a_2 \in [T + (J(\mathcal{H})\cap soc(\mathcal{H}))\ _R: \mathcal{H}]$, we get $b_1a_2 \in [T + (J(\mathcal{H})\cap soc(\mathcal{H}))\ _R: \mathcal{H}]$. Since $(a_1+b_1)(a_2+b_2)\mathcal{L} \subseteq T$ and $(a_1+b_1)\mathcal{L} \nsubseteq T + (J(\mathcal{H})\cap soc(\mathcal{H}))$ and $(a_2+b_2)\mathcal{L} \nsubseteq T + (J(\mathcal{H})\cap soc(\mathcal{H}))$, we have $(a_1+b_1)(a_2+b_2) = a_1a_2 + a_1b_2 + b_1a_2 + bb_2 \in [T + (J(Q)\cap soc(Q))\ _R: Q]$. But $a_1a_2, b_1a_2\ a_1b_2 \in [T + (J(Q)\cap soc(Q))\ _R: Q]$, so we get $bb_2 \in [T + (J(Q)\cap soc(Q))\ _R: Q]$ which is a contradiction. Consequently $I\mathcal{L} \subseteq T + (J(Q)\cap soc(Q))$ or $J\mathcal{L} \subseteq T + (J(Q)\cap soc(Q))$.

(⇐) $uv\mathcal{L} \subseteq T$, for $u\,, v \in R$, $\mathcal{L}$ is submodule of Q, that is $(u)\ (v)\ \mathcal{L} \subseteq T$, hence by hypothesis either $(u)\mathcal{L} \subseteq T + (J(Q)\cap soc(Q))$ or $(v)\mathcal{L} \subseteq T + (J(Q)\cap soc(Q))$ or $uv \in [T + (J(Q)\cap soc(Q))\ _R: Q]$. That is either $u\mathcal{L} \subseteq T + (J(Q)\cap soc(Q))$ or $v\mathcal{L} \subseteq T + (J(Q)\cap soc(Q))$ or $uv \in [T + (J(Q)\cap soc(Q))\ _R: Q]$. Therefore T is STPN-2-A submodule of Q.

From the proposition, we get the following result.

Corollary

A propersubmodule T of an R_ module Q is STPN-2-Asubmodule of Q if and only if $I\,Je \subseteq T$ for some ideals I, J of R and e$\in Q$, implies that either $Ie \subseteq T + (J(Q)\cap soc(Q))$or $Je \subseteq T + (J(Q)\cap soc(Q))$ or $IJ \subseteq [T + (J(Q)\cap soc(Q))_R: Q]$.

Lemma[7, lemma (2. 315)]

Let $\mathcal{L}$, $\mathbb{Q}$ and $\mathcal{B}$ be submodule of an R_module Q with $\mathbb{Q} \subseteq \mathcal{B}$. Then $(\mathcal{L} + \mathbb{Q}) \cap \mathcal{B} = (\mathcal{L} \cap \mathcal{B}) + \mathbb{Q} = (\mathcal{L} \cap \mathcal{B}) + (\mathbb{Q} \cap \mathcal{B})$.

Proposition

Let T, $\mathcal{L}$ be STPN _ 2 _Absorbing submodule of an R _ module Q with $\mathcal{L}$ is $\nsubseteq T$ and either $J(Q)\cap soc(Q) \subseteq T$ or $J(Q)\cap soc(Q) \subseteq \mathcal{L}$. Then $T \cap \mathcal{L}$ is STPN-2-A submodule of Q.

Proof

$T \cap \mathcal{L}$ is a proper submodule of $\mathcal{L}$ and $\mathcal{L}$ is a proper submodule of Q, implies that $T \cap \mathcal{L}$ is a proper submodule of Q. Suppose $\mathrm{J}(Q)\cap \mathrm{soc}(Q) \subseteq \mathcal{L}$ and $\mathrm{J}(Q)\cap \mathrm{soc}(Q) \nsubseteq \mathcal{F}$.Let uvA $\subseteq T \cap \mathcal{L}$ for u , v $\in$ R, A is a submodule of Q, it follows that uvA $\subseteq T$ and uvA $\subseteq \mathcal{L}$. But T, $\mathcal{L}$ are STPN-2-A submodule of Q, then either uA $\subseteq T + (\mathrm{J}(Q)\cap \mathrm{soc}(Q))$ or vA $\subseteq T + \mathrm{J}(Q)\cap \mathrm{soc}(Q))$ or uv$Q \subseteq T + (\mathrm{J}(Q)\cap \mathrm{soc}(Q))$ and uA $\subseteq \mathcal{L} + (\mathrm{J}(Q)\cap \mathrm{soc}(Q))$ or vA $\subseteq \mathcal{L} + (\mathrm{J}(Q)\cap \mathrm{soc}(Q))$ or uv$\mathcal{L} \subseteq T + (\mathrm{J}(Q)\cap \mathrm{soc}(Q))$. Thus either uA $\subseteq (T + (\mathrm{J}(Q)\cap \mathrm{soc}(Q))) \cap (\mathcal{L} + (\mathrm{J}(Q)\cap \mathrm{soc}(Q))$ or vA $\subseteq (T + (\mathrm{J}(Q)\cap \mathrm{soc}(Q))) \cap (\mathcal{L} + (\mathrm{J}(Q)\cap \mathrm{soc}(Q)))$ or uv$\mathcal{H} \subseteq (T + (\mathrm{J}(Q) \cap \mathrm{soc}(Q)\,)) \cap (\mathcal{L} + (\mathrm{J}(Q) \cap \mathrm{soc}(Q)\,))$. But $\mathrm{J}(Q) \cap \mathrm{soc}(Q) \subseteq \mathcal{L}$ then $\mathcal{L} + (\mathrm{J}(Q) \cap \mathrm{soc}(Q)\,) = \mathcal{L}$, it follows that either uA$\subseteq T + (\mathrm{J}(Q) \cap \mathrm{soc}(Q)\,) \cap \mathcal{L}$ or vA$\subseteq T + (\mathrm{J}(Q) \cap \mathrm{soc}(Q)\,) \cap \mathcal{L}$ or uv$Q \subseteq T + (\mathrm{J}(Q) \cap \mathrm{soc}(Q)\,) \cap \mathcal{L}$. By a bove lemma either uA$\subseteq (T \cap \mathcal{L}\,) + (\mathrm{J}(Q) \cap \mathrm{soc}(Q)\,)$ or vA$\subseteq (T \cap \mathcal{L}\,) + (\mathrm{J}(Q) \cap \mathrm{soc}(Q)\,)$ or uv$\mathcal{H} \subseteq (T \cap \mathcal{L}\,) + (\mathrm{J}(Q) \cap \mathrm{soc}(Q)\,)$. Therefore $T \cap \mathcal{L}$ is a STPN-2-A submodule of Q.

Lemma[9, remark, p 14]

If Q is a faithful multiplication R_ module, then $J(Q) = \mathrm{J(R)}Q$.

Lemma [10, *coro.* (2. 14)(*i*)]

Let Q be a faithful multiplication R_ module, $soc(Q) = soc(R)Q$

Proposition (2 . 11)

Let Q be a faithful multiplication R_module and T be a proper submdule of Q. Then T is STPN-2-A submodule of Q if and only if $[T:_R Q]$ is STPN-2-A ideal of R.

Proof

(⟹)Let $I_1\, I_2\, I_3 \subseteq [T:_R Q]$for I_1 , I_2 , I_3 are ideals ofR , it follows that $I_1\, I_2\, I_3\, Q \subseteq T$. But Q is multiplication, then $I_1\, I_2\, I_3 Q = K_1\, K_2\, K_3 \subseteq T$, by taking $K_1 = I_1 Q$, $K_2 = I_2\, Q$ and $K_3 = I_3\, Q$. But T is a STPN-2-A then either $K_1\, K_3 \subseteq T + (J(Q) \cap soc(Q)\,)$ or $K_2\, K_3 \subseteq T + (J(Q) \cap soc(Q)\,)$ or $K_1\, K_2 \subseteq T + (J(Q) \cap soc(Q)\,)$. But Q is a faithful multiplication, then by a bove Lemmas $Toc(\,Q) = Toc(\,R\,)Q$, and $J(\,Q\,) = J(\,R\,)Q$. Thus either $I_1\, I_3\, Q \subseteq [T:_R Q]Q + (J(R)Q \cap soc(R)Q)$ or $I_2\, I_3\, Q \subseteq [T:_R Q]Q + (J(R)Q \cap soc(R)Q)$ or $I_1\, I_2\, Q \subseteq [T:_R Q]Q + (J(R)Q \cap soc(R)Q)$. That is either $I_1\, I_3 \subseteq [T:_R Q] + (J(R) \cap soc(R))$ or $I_2\, I_3 \subseteq [T:_R Q] + (J(R) \cap soc(R))$ or $I_1\, I_2 \subseteq [T:_R Q] + \big(J(R) \cap soc(R)\big) = \Big[[T:_R Q] + \big(J(R) \cap soc(R)\big):_R R\Big]$. Therefore $[T:_R Q]$ is STPN-2-A ideal of R .

(⟸) Let $I_1\, I_2\, K \subseteq T$ for I_1 , I_2 are ideals of R, and $K \subseteq Q$. Since Q is a multiplication, then $K = I_3\, Q$ for some ideal I_3 of R. Thatis $I_1\, I_2\, I_3 Q \subseteq T$, it follows that $I_1\, I_2\, I_3 \subseteq [T:_R Q]$. But $[T:_R Q]$ is a STPN-2-A ideal of R, then either $I_1\, I_3 \subseteq [T:_R Q] + (J(R) \cap soc(R))$ or $I_2\, I_3 \subseteq [T:_R Q] + (J(R) \cap soc(R))$ or $I_1\, I_2 \subseteq [T:_R Q] + \big(J(R) \cap soc(R)\big) = \Big[[T:_R Q] + \big(J(R) \cap soc(R)\big):_R R\Big]$. Thus either $I_1\, I_3 Q \subseteq [T:_R Q]Q + (J(R)Q \cap soc(R)Q)$ or $I_2\, I_3\, Q \subseteq [T:_R Q]Q + (J(R)Q \cap soc(R)Q)$ or $I_1\, I_2\, Q \subseteq [T:_R Q]Q + (J(R)Q \cap soc(R)Q)$. Since Q is a faithful multiplication, then by a bove Lemmas $Toc(\,Q\,) = Toc(\,R\,)Q$, and then $J(\,Q) = J(\,R\,)Q$. It follows that either $I_1\, K \subseteq T + (J(Q) \cap soc(Q)\,)$ or $I_2\, K \subseteq T + (J(Q) \cap soc(Q)\,)$ or $I_1\, I_2 \subseteq [T + (J(Q) \cap soc(Q)\,) :_R Q]$. Therefore T is STPN-2-A submodule of Q.

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شبه زائف بقوة مايقرب من 2-وحدات فرعية ماصة

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معلومات المؤلف

الايميل:
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في هذا العمل قمنا بدراسة مفهوم شبه قوي لما يقرب من 2-وحدات فرعية ماصة وبعض التعميم, وهو تعميم من 2-وحدات فرعية ماصة وشكل قوي لما يقرب من 2-وحدات فرعية ماصة و 2-زائفة ماصة. يتم تقديم توصيف وامثلة على الاقتراح, بالاضافة الى العديد من خصائص الاقتراح المختلفة التي تم اثباتها.