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A Double Intuitionistic Compact Space in Intuitionistic Topological Spaces

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Abstract

The aim of this paper is to construct the basic concept related to compactness in Double intuitionistic topological spaces (X,Ψ). We introduced the concepts of the Double intuitionistic compact and some of its types. Also, we found the relationships between these types with proofs and examples.

Introduction:

The concept of general topological spaces, their types, and basic concepts were introduced by Step by Step [1]. The theory of intuitionistic fuzzy sets was examined and developed more on fuzzy sets [2,3]. The concept was used to define intuitionistic sets and on intuitionistic gradation of openness by Coker [4,5,6].

In [7] presented some types of compactness in double topological spaces. Generalization of the concept of double set was first introduced by Kandil, Tantawy, and Wafaie on flou intuitionistic topological spaces and compact space [8,9]. Also introduced the concept of generalized closed sets in topology and investigated basic properties of generalization closed set by Levine [10]. He explains pairwise compact in intuitionistic double topological spaces in [11]. After presentation of the operations on submaximality intuitionistic topological spaces [12]. The fuzzy concept has invaded almost all branches of mathematics ever since the introduction of fuzzy sets by Zadeh [13]. The goal of this paper is to introduce a new class of space in (DITS) namely Double intuitionistic compact space which is between the class of Double intuitionistic open cover, Double intuitionistic compact subspace and Double intuitionistic finite intersection property, we also study the basic characteristics and qualities related to these types and the relationships between them.

Preliminaries

We recall the following definitions, which are needed, in our work.

Definition 2.1 [4],[5] Let $X \neq \emptyset$, and let B and D be intuitionistic set (IS, for short) having the form $B = \langle x, B_1, B_2 \rangle, D = \langle x, D_1, D_2 \rangle$ respectively. Also, $\{B_i : i \in I\}$ be an arbitrary family of IS in X , where $B_i = \langle x, {B_i}^{(1)}, {B_i}^{(2)} \rangle$, afterward:

1) $\widetilde{\emptyset} = \langle x, \emptyset, X \rangle$; $\widetilde{X} = \langle x, X, \emptyset \rangle$.

2) $B \subseteq D$ *iff* $B_1 \subseteq D_1$ *and* $D_2 \supseteq B_2$.

3) $B^{\,c} = \langle x, B_2, B_1 \rangle$.

4) $\cup B_i = \langle x, \cup B_i^{(1)}, \cap B_i^{(2)} \rangle$, $\cap B_i = \langle x, \cap B_i^{(1)}, \cup B_i^{(2)} \rangle$.

5) $B - D = B \cap D^c$.

6) $B = D$ *if and only if* $B \subseteq D$ *and* $D \subseteq B$.

Definition 2.2 [4],[6] Let X be a non-empty set, an intuitionistic set B (IS, for short) is an object having g the form $B = \langle x, B_1, B_2 \rangle$ where B_1 and B_2 are disjoint subset of X . Then B_1 is called set of members of B, while B_2 is called set of nonmembers of B.

Definition 2.3 [12],[2] An intuitionistic topology (IT, for short) on a non-empty set X, is a family T of IS in X containing $\widetilde{\emptyset}, \widetilde{X}$ and closed under arbitrary unions and finitely intersections. The pair (X, T) is called ITS.

Definition 2.4 [9] Let $X \neq \emptyset$.

1) A Double-set (D- set, for short) $\underline{A}$ is an ordered pair $(A_1, A_2) \in \wp(X) \times \wp(X)$ such that $A_1 \subseteq A_2$.

2) $D(X) = \{(A_1, A_2) \in \wp(X) \times \wp(X), A_1 \subseteq A_2\}$ is the family of all D-sets on X .

3) The D-set $\underline{X} = (X, X)$ is called the universal D-set, and the D-set $\underline{\emptyset} = (\emptyset, \emptyset)$ is called the empty D-set.

4) Let $\eta_1, \eta_2 \subseteq \wp(X)$. The product of η_1 and η_2, denoted by $\eta_1 \times \eta_2$ defined by $\eta_1 \times \eta_2 = \{(A1, A2) : A_1 \in \eta_1, A_2 \in \eta_2, A_1 \subseteq A_2\}$.

5) Let $\underline{A} = (A_1, A_2)$; $\underline{G} = (G_1, G_2) \in D(X)$: 1) $(\underline{A^c}) = (A^c_2, A^c_1)$ where A^c is the complement of $\underline{A}$.

2) $\underline{A} - \underline{G} = (A_1 - G_2, A_2 - G_1)$

6) The double set $\underline{A} = ((A_1, A_2)$ is said to be a finite double set if A_2 is finite set.

7) If $\{\underline{A_\alpha} : \alpha \in \Lambda\} \subseteq D(X)$ such that $\underline{A_\alpha} = (A_{1\alpha}, A_{2\alpha})$, then $\cup_{\alpha \in \Lambda} \underline{A_\alpha} = (\cup_{\alpha \in \Lambda} A_{1\alpha}, \cup_{\alpha \in \Lambda} A_{2\alpha})$ and $\cap_{\alpha \in \Lambda} \underline{A_\alpha} = (\cap_{\alpha \in \Lambda} A_{1\alpha}, \cap_{\alpha \in \Lambda} A_{2\alpha})$.

Definition 2.5 [9] Let X be a non-empty set. The family η of D-sets in X is called a double topology on X if it satisfies the following axioms:

a) $\underline{\emptyset}$, $\underline{X} \in \eta$.

b) If $\underline{A}$, $\underline{G} \in \eta$, then $\underline{A} \cap \underline{G} \in \eta$,

c) If $\{\underline{Az} : z \in Z\} \subseteq \eta$, then $\cup_{z \in Z} \underline{Az} \in \eta$. The pair (X, η) is called a DTS. Each element of η is called an open D-set in X . The complement of open D-set is called closed D-set.

Definition 2.6 [9] Let X be a non-empty set defined by:

1) $IN(X) = \{\underline{\emptyset}, \underline{X}\}$, then IN is a Double topology on X and is called indiscrete Double topology. (X, IN) is called indiscrete Double space.

2) $dis(X) = \wp(X) \times \wp(X)$ (power set of X's), then dis is a Double topology on X and is called discrete Double topology. (X, dis) is called discrete Double space.

Definition 2.7 [1] A topological space (X, η) is said to be compact space if every open cover of X has a finite sub-cover.

Definition 2.8 [8] Let (X, η) be a double topological space and let $\underline{A} \in D(X)$. A collection $\underline{O} = \{\underline{G\alpha} : \alpha \in \Lambda\} \subseteq D(X)$ is said to be a double cover (D-cover, for Short) of A if $\underline{A} \subseteq \cup_{\alpha \in \Lambda} G_\alpha$. If $\underline{O} \subseteq \eta$, then $\underline{O}$ is called double open cover (D-open cover, for short).

Definition 2.9 [8] A double topological space (X, η) is said to be a CD-compact space if for every double closed set $\underline{F}$ and for every D-open cover $\underline{O}$ of $\underline{F}$ has a finite sub-cover.

Definition 2.10 [7], [11] A double topological space (X, η) is said to be D-compact space if every D-open cover of X has a finite sub-cover.

Definition 2.11 [9] Let (X, η) be a DTS and Y be a non-empty subset of X. Then, $\eta_Y = \{\underline{q} \cap \underline{Y}: \underline{q} \in \eta \text{ and } \underline{Y} = (Y, Y)\}$ is a double topology on Y. The DTS (Y, η_Y) is called a double topological subspace of (X, η) (DT-subspace, for short).

Definition 2.12 [7] Let (X, η) be a double topological space. The collection $V = \underline{H\alpha :}\ \alpha \in \Lambda\} \subseteq D(X)$ is said to have the finite intersection property (FIP, for short) if for every finite sub-collection $\underline{\{H\alpha_i} : i = 1, 2\ldots, n\}$ of A, we have $\cap_{i=1}^{n} H_{\alpha i} \neq \underline{\emptyset}$.

Definition 2.13 [4] Let X nonempty set, $W \in X$ a fixed element in X , and let $M = \langle x, M_1, M_2\rangle$ be an intuitionistic set (IS, for short). The IS $\dot{W}$ defined by $\dot{W} = \langle x, \{W\}, \{W\}^c\rangle$ is called an intuitionistic point (IW, for short) in X.

3- Double intuitionistic compact space in DITS

In this part, we introduce and study the idea of a new type of Double compactness defined in terms of Double intuitionistic topological spaces$(X.\Psi)$. Calling Double intuitionistic compactness. Finally; we investigate its results with compactness among other things.

We start this section by the following definitions.

Definition 3.1 Let X be a non-empty set.

1) A Double intuitionistic set (Double I-set, for short) is an ordered pair $(Q, D) = (\langle x, Q_1, Q_2\rangle, \langle x, D_1, D_2\rangle) \in pI(X) \times pI(X)$ such that $Q \subseteq D$.
2) Double $I(X) = \{(Q, D) \in pI(X) \times pI(X), Q \subseteq D\}$ is the family of all Double I-sets on X.
3) Let $\Psi_1, \Psi_2 \subseteq pI(X)$. The Double product of Ψ_1 and Ψ_2, defined by $\Psi_1 \times \Psi_2 = \{(Q, D): Q \in \Psi_1, D \in \Psi_2, Q \subseteq D\}$.
4) The Double I-set (Q, D) is said to be a finite Double I-set if $\mathcal{D}$ is finite Double I-set.
5) The Double I-set $(\langle x, X, \emptyset\rangle, \langle x, X, \emptyset\rangle) = (\tilde{X}, \tilde{X})$ is called the universal Double I-set, and the Double I- set, $(\tilde{\emptyset}, \tilde{\emptyset}) = (\langle x, \emptyset, X\rangle, \langle x, \emptyset, X\rangle)$ is called the empty Double I-set.
6) Let $(Q, D), (C, H) \in$ Double I(X):
    1) $(Q, D)^c = (D^c, Q^c) = (\langle x, D_1, D_2\rangle^c, \langle x, Q_1, Q_2\rangle^c) = (\langle x, D_2, D_1\rangle, \langle x, Q_2, Q_1\rangle)$.
    2) $(Q, D) \setminus (C, H) = ((Q \setminus H), (D \setminus C)) = (\langle x, Q_1, Q_2\rangle, \langle x, D_1, D_2\rangle) \setminus (\langle x, C_1, C_2\rangle, \langle x, H_1, H_2\rangle) = (\langle x, Q_1, Q_2\rangle \setminus \langle x, H_1, H_2\rangle), (\langle x, D_1, D_2\rangle \setminus \langle x, C_1, C_2\rangle)$.
7) If $\{(Q_{1\alpha}, Q_{2\alpha}) : \alpha \in \Lambda\} \subseteq \text{Double } I(X)$, then $\cup_{\alpha \in \Lambda} (Q_{1\alpha}, Q_{2\alpha}) = < x, \cup_{\alpha \in \Lambda} Q_{1\alpha}, \cap_{\alpha \in \Lambda} Q_{2\alpha} >$ and $\cap_{\alpha \in \Lambda} (Q_{1\alpha}, Q_{2\alpha}) = < x, \cap_{\alpha \in \Lambda} Q_{1\alpha}, \cup_{\alpha \in \Lambda} Q_{2\alpha} >$. Each element of Ψ is called a Double intuitionistic open set (DIOS, for short) in X. The complement of DIOS is called a Double intuitionistic closed set (DICS, for short).

Now, we want to introduce the next important theorem to construct the Double intuitionistic topological spaces**.**

Theorem 3.2 Let $X \neq \emptyset$,then the family T of all Double intuitionistic open sets in X is Double intuitionistic topological spaces (DITS)**.**

Proof Let (X, T) be intuitionistic topological spaces (ITS), then:

1) $\tilde{\emptyset} = \langle x, \emptyset, X\rangle$, $\tilde{X} = \langle x, X, \emptyset\rangle \in IT$, $(\tilde{\emptyset}, \tilde{\emptyset}), (\tilde{X}, \tilde{X}) \in$ DITS.

2) Let $(Q, D), (C, H) \in DIT$, $Q, D, C, H \in IT$. Since IT is intuitionistic topology, then $Q \cap D \in IT$ and $C \cap H \in IT$. Now, let $\mathcal{K} = (Q, D)$ and $\mathcal{W} = (C, H)$, $(\mathcal{K}, \mathcal{W}) = ((Q, C), (D, H)) \in$ DITS.

3) Let (Q_s, D_s) be a family of IS and s ∈ S and $(Q_s, D_s) \in D\Pi$, $Q_s, D_s \in$ ITS, since IT is intuitionistic topology, then $\cup_{s\in S} Q_s \in \Pi$ and $\cup_{s\in S} D_s \in \Pi$. Thus $\cup_{s\in S} (Q_s, D_s) \in D\Pi$. Therefore, (X, T) is Double intuitionistic topological spaces.

Definition 3.3 Let (X , Ψ) be a DITS. A family $(O_1, O_2) = \{(G_{1\alpha}, G_2) : \alpha \in \Lambda\} \subseteq$ Double I(X) is said to be a Double cover of X if $(\tilde{X},\tilde{X}) = \cup_{\alpha\in\Lambda} (G_{1\alpha}, G_{2\alpha})$. If (O_1, O_2) is finite and Double cover X, then (O_1, O_2) is called a finite Double cover of X. If $(O_1, O_2) \subseteq \Psi$, then (O_1, O_2) is called Double intuitionistic open cover (Double I-open cover, for short). If $(O_1, O_2) \subseteq \Psi^c$, *then* (O_1, O_2) is called Double intuitionistic closed cover (Double I-closed cover, for short).

Definition 3.4 Let $(O_1, O_2) = \{(G_{1\alpha}, G_{2\alpha}) : \alpha \in \Lambda\}$ be a Double cover of X and $\{(B_{1j}, B_{2j}) : j \in J\}$ be a subfamily of (O_1, O_2) and Double cover X, then $\{(B_{1j}, B_{2j}) : j \in J\}$ is called sub cover from (O_1, O_2).

Definition 3.5 A Double intuitionistic topological spaces (X , Ψ) is called to be Double intuitionistic compact space (Double I-compact, for short) if every Double I-open cover $(O_1, O_2) = \{(G_{1\alpha}, G_{2\alpha}) : \alpha \in \Lambda\}$ of X has a finite sub cover.

Example 3.6 Let $X = \{m,n,r\}$; $\Psi = \{(\tilde{\emptyset},\tilde{\emptyset}),(\tilde{X},\tilde{X}),(V_1,V_1),(V_2,V_2)\}$ where $(V_1,V_1) = (\langle x,\{m\},\{n,r\}\rangle, \langle x,\{m\},\{n,r\}\rangle)$ and $(V_2,V_2) = \langle x,\{n,r\},\{m\}\rangle, \langle x,\{n,r\},\{m\}\rangle)$.Take $C_1 = \{(V_1,V_1),(V_2,V_2)\}$ is Double I-open cover for X and it's a finite sub cover of X , so this Double cover satisfy the definition of Double I-compact space. Now, we introduce all Double I-open cover for X as follows : $C_2 = \{(\tilde{\emptyset},\tilde{\emptyset}), (\tilde{X},\tilde{X}), (V_1,V_1)\}$;$C_3 = \{(\tilde{\emptyset},\tilde{\emptyset}), (\tilde{X},\tilde{X}), (V_2,V_2)\}$; $C_4 = \{(\tilde{\emptyset},\tilde{\emptyset}),(\tilde{X},\tilde{X}),(V_1,V_1),(V_2,V_2)\} = \Psi$; $C_5 = \{(\tilde{X},\tilde{X}),(V_1,V_1)\}$; $C_6 = \{(\tilde{X},\tilde{X}),(V_2,V_2)\}$; $C_7 = \{(\tilde{X},\tilde{X}),(V_1,V_1),(V_2,V_2)\}$; $C_8 = \{(\tilde{\emptyset},\tilde{\emptyset}),(\tilde{X},\tilde{X})\}$; $C_9 = \{(\tilde{X},\tilde{X})\}$. In addition, every Double cover of them has a finite sub cover. Hence (X , Ψ) is Double I-compact.

Remark 3.7

1) If Ψ is finite set, then (X, Ψ) is Double I-compact space, so every Double I-open cover has a finite sub cover. Special case: (X, IN) is Double I-compact space for any X (finite or in finite).

2) If Ψ is infinite set, then (X, dis) is not Double I-compact space, since the Double I-open cover $(O_1, O_2) = \{(\tilde{p},\tilde{p});(\tilde{p},\tilde{p}) \in X\}$ has no finite sub cover. If X is finite, then (X, dis) is Double I-compact space (by remark (1)).

Proposition 3.8 Let (X, Ψ) be a Double I-compact space. Then every Double I-closed set is Double I-compact.

Proof Let (X,Ψ) be a Double I-compact space, (Q, D) be a Double I-closed set and let the collection $(O_1, O_2) = \{(G_{1\alpha}, G_{2\alpha}) : \alpha \in \Lambda\} \subseteq \Psi$ be a Double I-open cover of (Q, D), *so* $(Q, D) \subseteq \cup_{\alpha\in\Lambda} (G_{1\alpha}, G_{2\alpha})$. Since $(Q, D)^c$ is Double I-open set, so that $(\tilde{X},\tilde{X}) = (Q,D) \cup (Q, D)^c \subseteq \cup_{\alpha\in\Lambda} (G_{1\alpha}, G_{2\alpha}) \cup (Q, D)^c$, *so* $(\tilde{X},\tilde{X}) = \cup_{\alpha\in\Lambda} (G_{1\alpha}, G_{2\alpha}) \cup (Q, D)^c$. Since $(G_{1\alpha}, G_{2\alpha}) \in \Psi$, $\forall \alpha \in \Lambda \wedge (Q, D)^c \in \Psi$ (since (Q, D) is Double I-closed set) , $\{(G_{1\alpha}. G_{2\alpha}) : \alpha \in \Lambda\} \cup \{(Q, D)^c\}$ is Double I-open cover of X. Since (X, Ψ) is Double I-compact, thus there exist $\alpha_1, \alpha_2, \dots, \alpha_n$; $(\tilde{X},\tilde{X}) = (\cup_{j=1}^{n} (G_{1\alpha j}, G_{2\alpha j})) \cup \{(Q, D)^c\}$. *But* $(Q, D) \subseteq X$, $(Q, D) \subseteq (\cup_{j=1}^{n} (G_{1\alpha j}, G_{2\alpha j})) \cup \{(Q, D)^c\}$. Since $(Q,D) \cap (Q, D)^c = (\tilde{\emptyset},\tilde{\emptyset})$, $(Q, D) \subseteq (\cup_{j=1}^{n} (G_{1\alpha j}, G_{2\alpha j})$.

Therefore (Q, D) is Double - compact.

The converse of the proposition may not be true in general as shown by the following example:

Example 3.9 Let $X = \{1,3\}$; $\Psi = \{(\tilde{\emptyset},\tilde{\emptyset}),(\tilde{X},\tilde{X}),(K_1,\tilde{X}),(K_1^c,K_1^c),(\tilde{\emptyset},K_1^c)\}$ *where* $(K_1,\tilde{X}) = (\langle x,\{1\},\{3\}\rangle, \langle x,X,\emptyset\rangle)$, $(K_1^c,K_1^c) = (\langle x,\{3\},\{1\}\rangle, \langle x,\{3\},\{1\}\rangle)$

and $(\widetilde{\emptyset}, K_1^c)$=$\langle x, \emptyset, X\rangle, \langle x, \{3\}, \{1\}\rangle)$. Take $C_1 = \{(K_1, \tilde{X}), (K_1^c, K_1^c)\}$ is Double I-open cover for X and it is a finite sub cover. Then (X, Ψ) is Double I-compact. Now, let (K_1^c, K_1^c) has a Double I-compact, but it's not Double I-closed, because, $(K_1^c, K_1^c)^c =$ $(\langle x, \{1\}, \{3\}\rangle, \langle x, \{1\}, \{3\}\rangle) \notin \Psi$.

Theorem 3.10 Let (X,Ψ) be DITS. Then (X,Ψ) is Double I-compact if and only if (X,T_1) is I-compact where $T_1 = \{Q: (Q, D) \in \Psi\}$.

Proof Let (X, Ψ) be a Double I-compact space and let $O_1 = \{(G_1: \alpha \in \Lambda\} \subseteq T_1$be an I-open cover of X or $\tilde{X} = \cup_{\alpha \in \Lambda} G_{1\alpha}$. Then the family $(O_1, O_2) = \{(G_{1\alpha}, G_{2\alpha}): G_{2\alpha} \in T_2; \alpha \in \Lambda\}$ is a Double I-open cover of X. Since (X, Ψ) is Double I-compact, then there exists a finite sub cover $\{(G_{1\alpha j}, G_{2\alpha j}): j = 1, 2 \ldots n\} \subseteq (O_1, O_2)$ of X, so $(\tilde{X}, \tilde{X}) = \cup_{j=1}^{n} (G_{1\alpha j}, G_{2\alpha j})$. So $\{g_{1\alpha j} : j = 1, 2 \ldots, n\} \subseteq O_1$ is a finite sub cover of X. Thus (X,T_1) is a T_1 I-compact.

Conversely: Let (X,T_1) is an T_1 I-compact, let $(O_1, O_2) = \{(G_{1\alpha}, G_{2\alpha}): \alpha \in \Lambda\} \subseteq \Psi$ be a Double I-open cover of X, so $(\tilde{X}, \tilde{X}) = \cup_{\alpha \in \Lambda} (G_{1\alpha}, G_{2\alpha})$. Since for every $(G_{1\alpha}, G_{2\alpha})$ in (O_1, O_2) there exist $G_{1\alpha}, G_{2\alpha} \in \wp I(X)$ and $G_{1\alpha} \subseteq G_{2\alpha}$ such that $(G_{1\alpha}, G_{2\alpha})$. Then $\tilde{X} = \cup_{\alpha \in \Lambda} G_{1\alpha}$ the collection $O_1 = \{G_{1\alpha}: (G_{1\alpha}, G_{2\alpha}) \in (O_1, O_2)\} \subseteq O_1$ is a T_1 I-open cover of X, but (X, T_1) is T_1 I-compact, then there exists a finite sub cover $\{G_{1\alpha\, j} : j = 1, 2 \ldots, n\} \subseteq O_1$ such that $\tilde{X} = \cup_{j=1}^{n} G_{1\alpha\ j}$. Now since $G_{1\alpha j} \subseteq G_{2\alpha j}$ then $\cup_{j=1}^{n} G_{1\alpha j} \subseteq \cup_{j=1}^{n} G_{2\alpha j}$. Hence $(\tilde{X}, \tilde{X}) = (\cup_{j=1}^{n} G_{1\alpha j}, \cup_{j=1}^{n} G_{2\alpha j}) = \cup_{j=1}^{n} (G_{1\alpha j}, G_{2\alpha j})$. Therefore (X,Ψ) is Double I-compact.

Theorem 3.11 Let (X,Ψ) be DITS. Then (X,Ψ) is Double I-compact, then (X,T_3) is I-compact where $T_3 = \{Q : (Q, X) \in \Psi\}$.

Proof Let (X, Ψ) be a Double I-compact and let $O = \{(G_\alpha: \alpha \in \Lambda, (G_\alpha, X) \in \Psi\} \subseteq T_3$ be an I-open cover of X, so $\tilde{X} = \cup_{\alpha \in \Lambda} G_\alpha$. Then $(\tilde{X}, \tilde{X}) = \cup_{\alpha \in \Lambda} (G, X)$. Thus, the collection $(O_1, O_2) = \{(G_\alpha, X): G \in O\}$ is a Double I-open cover of X. But (X, Ψ) is Double I-compact, then there exist a finite sub cover $\{(G_{\alpha j}, X): j = 1, 2 \ldots, n\} \subseteq (O_1, O_2)$ of X, so $(\tilde{X}, \tilde{X}) = \cup_{j=1}^{n} (G_{\alpha j}, X)$, $\tilde{X} = \cup_{j=1}^{n} G_{\alpha j}$. Therefore (X,T_3) is I-compact.

Theorem 3.12 Let (X,Ψ) be DITS. Then

a) $\Psi_1 = \{(Q, Q), (Q, D) \in \Psi\}$.

b) $\Psi_2 = \{(D, D), (Q, D) \in \Psi\}$ are Double intuitionistic topologies on X.

Proof a) Since $(\widetilde{\emptyset}, \widetilde{\emptyset}), (\tilde{X}, \tilde{X}) \in \Psi, (\widetilde{\emptyset}, \widetilde{\emptyset}), (\tilde{X}, \tilde{X}) \in \Psi_1$. Let $((Q, Q), (U, U)) \in \Psi 1$, so there exist $D, V \in pI(X)$ such that $(Q, D), (U, V) \in \Psi$, $(Q \cap U, D \cap V) \in \Psi$, $(Q \cap U, Q \cap U) \in \Psi_1$. Thus $(Q, Q) \cap (U, U) \in \Psi_1$, let $\{(Q_\alpha, Q_\alpha): \alpha \in \Lambda\} \subseteq \Psi_1$, so for each Q_α there exist $Q_\alpha \in pI(X)$ such that $(Q_{\alpha,}, D_\alpha) \in \Psi, \{(Q_{\alpha,}, D_\alpha): \alpha \in \Lambda\} \subseteq \Psi$. Hence $\cup_{\alpha \in \Lambda} (Q_\alpha, D_\alpha) \subseteq \Psi$. So $\cup_{\alpha \in \Lambda} (Q_\alpha, D_\alpha) \subseteq \Psi_1$. Therefore Ψ_1 is Double I topology on X.

b) AS same way (a).

Theorem 3.13 Let (X, Ψ) be DITS. Then (X,Ψ) is an Ψ-Double I-compact if and only if (X, Ψ_1) is a Ψ_1 Double I-compact.

Proof Let (X, Ψ) be an Ψ-Double I-compact, let $(O_1, O_1) = \{(G_{1\alpha}, G_{1\alpha}): \alpha \in \Lambda\} \subseteq \Psi_1$be Ψ_1 Double I-open cover of X, so $(\tilde{X}, \tilde{X}) = \cup_{\alpha \in \Lambda} (G_{1\alpha}, G_{1\alpha})$. For each $G_{1\alpha}$ there exists $G_{2\alpha} \in pI(X)$ such that $(G_{1\alpha}, G_{1\alpha}) \subseteq (G_{1\alpha}, G_{2\alpha}) \in \Psi$, thus $(\tilde{X}, \tilde{X}) = \cup_{\alpha \in \Lambda} (G_{1\alpha}, G_{2\alpha})$. Hence the collection $\{(G_{1\alpha}, G_{2\alpha}): \alpha \in \Lambda\} \subseteq \Psi$ is an Ψ Double I-open cover of X, then there exist a finite sub cover $\{(G_{1\alpha}, G_{2\alpha\ j}): j = 1, 2 \ldots, n\} \subseteq \{(G_{1\alpha}, G_{2\alpha}): \alpha \in \Lambda\}$ of X, $(\tilde{X}, \tilde{X}) = \cup_{j=1}^{n} (G_{1\alpha j}, G_{2\alpha j})$, $\tilde{X} = \cup_{j=1}^{n} G_{1\alpha j}$, $(\tilde{X}, \tilde{X}) = \cup_{j=1}^{n} (G_{1\alpha j}, G_{1\alpha j})$. Therefore (X, Ψ_1) is a Ψ_1 Double I-compact.

Conversely: Let (X, Ψ_1) be a Ψ_1 Double I-compact, let $(O_1, O_2) = \{(G_{1\alpha}, G_{2\alpha}): \alpha \in \Lambda\} \subseteq \Psi$ be an Ψ Double I-compact, so $(\tilde{X},\tilde{X}) = \cup_{\alpha \in \Lambda}(G_{1\alpha}, G_{2\alpha})$. Then $\tilde{X} = \cup_{\alpha \in \Lambda} G_{1\alpha}$, $(\tilde{X},\tilde{X}) = \cup_{\alpha \in \Lambda}(G_{1\alpha}, G_{2\alpha})$. Thus the collection $\{(G_{1\alpha}, G_{1\alpha}): \alpha \in \Lambda\} \subseteq \Psi_1$ be an Ψ_1 Double I-open cover of X , then there exist a finite sub cover $\{(G_{1\alpha j}, G_{1\alpha j}) : j = 1, 2..., n\} \subseteq \{(G_{1\alpha}, G_{1\alpha}): \alpha \in \Lambda\}$ *of* X , $(\tilde{X},\tilde{X}) = \cup_{j=1}^{n} (G_{1\alpha j}, G_{1\alpha j})$, $(\tilde{X},\tilde{x}) = \cup_{j=1}^{n} (G_{1\alpha j}, G_{2\alpha j})$. Therefore, (X,$\Psi$) is an Ψ Double I-compact.

Theorem 3.14 Let (X, Ψ) be DITS. Then (X, Ψ_2) is a Ψ_2 Double I-compact if and only if (X, T_2) is an T_2 I-compact.

Proof Let (X, Ψ_2) be an Ψ_2 Double I-compact, let $O_2 = \{G_{2\alpha}: \alpha \in \Lambda\} \subseteq T_2$ be an T_2 I-open cover of X, so $\tilde{X} = \cup_{\alpha \in \Lambda} G_{2\alpha}$. *Then* $(\tilde{X},\tilde{X}) = \cup_{\alpha \in \Lambda}(G_{2\alpha}, G_{2\alpha})$ .Thus the collection $(O_2, O_2) = \{(G_{2\alpha}, G_{2\alpha}): G_{2\alpha} \in O_2\} \subseteq \Psi_2$ is an Ψ_2 Double I-open cover of X, then there exist a finite sub cover $\{(G_{2\alpha j}, G_{2\alpha j}): j = 1, 2, \dots n\} \subseteq \Psi_2$ *of* X*, so* $(\tilde{X},\tilde{X}) = \cup_{j=1}^{n} (G_{2\alpha j}, G_{2\alpha j})$, $\tilde{X} = \cup_{j=1}^{n} G_{2\alpha j}$. Therefore, (X, T_2) is a T_2 I-compact.

Conversely: Let (X,T_2) be a T_2 I-compact, let $(O_2, O_2) = \{(G_{2\alpha}, G_{2\alpha}): \alpha \in \Lambda\} \subseteq \Psi_2$ be an Ψ_2 Double I-open cover of X, so $(\tilde{X},\tilde{X}) = \cup_{\alpha \in \Lambda}(G_{2\alpha}, G_{2\alpha})$. Then $\tilde{X} = \cup_{\alpha \in \Lambda} G_{2\alpha}$. Thus the collection $\{G_{2\alpha} : (G_{2\alpha}, G_{2\alpha}) \in (O_2, O_2)\} \subseteq T_2$ be an T_2 I-open cover of X, so there exist a finite sub cover $\{G_{2\alpha j}: j = 1, 2..., n\} \subseteq \{G_{2\alpha} : (G_{2\alpha}, G_{2\alpha}) \in (O_2, O_2)\}$ *of* X , $\tilde{X} = \cup_{j=1}^{n} G_{2\alpha j}$, $(\tilde{X},\tilde{X}) = \cup_{j=1}^{n} (G_{2\alpha j}, G_{2\alpha j})$. Therefore, (X, Ψ_2) is a Ψ_2 Double I-compact.

Proposition 3.15 Let (X, Ψ_1), (X, Ψ_2) be two DITS such that Ψ_2 is finer than Ψ_1. If (X, Ψ_2) is Double I-compact, then (X, Ψ_1) is Double I-compact.

Proof It is clear by using theorem 3.14, we get the result.

Proposition 3.16 Let (X, Ψ) be a DITS, $(Q, D) \subseteq X$, if it was (C, H) is Double I-compact, (Q, D), is Double I-closed set, then $(C, H) \cap (Q, D)$ is Double I-compact.

Proof Suppose that $(O_1, O_2) = \{(G_{1\alpha}, G_{2\alpha}): \alpha \in \Lambda\}$ is Double I-open cover of $(C, H) \cap (Q, D)$, so $(C, H) \cap (Q, D) \subseteq \cup_{\alpha \in \Lambda}(G_{1\alpha}, g_{2\alpha})$, $(C, H) \subseteq (\cup_{\alpha \in \Lambda}(G_{1\alpha}, G_{2\alpha})) \cup (Q, D)^c$. Since (Q, D) is Double I- closed set of X's, then $(Q, D)^c$ is Double I-open set. Hence $\{(G_{1\alpha}, g_{2\alpha})\} \cup \{(Q, D)^c\}$ is Double I-open cover of (C, H). But (C, H) is Double I-compact, so there exist a finite sub cover $\{(G_{1\alpha j}, G_{2\alpha j}): j = \{1, 2..., n\} \cup \{(Q, D)^c\}$ *of* (C, H), $(C, H) \subseteq (\cup_{j=1}^{n} (G_{1\alpha j}, G_{2\alpha j})) \cup \{(Q, D)^c\}$, $(C, H) \cap (Q, D) \subseteq \cup_{j=1}^{n} (G_{1\alpha j}, G_{2\alpha j})$. Therefore $(C$, H) ∩ (Q, D) is Double I-compact.

Definition 3.17 Let (X,Ψ) be DITS, and Y be a sub space of X. Then Y is called Double I-compact if every Double I-open cover from X Double cover Y has a finite sub cover.

Notes: If Y is Double I-compact subset of (X,Ψ), then Y, Ψ_Y) is called Double I-compact sub space of (X, Ψ).

Example 3.18 Let $X = \{x, y, z\}$; $\Psi = \{(\tilde{\emptyset},\tilde{\emptyset}), (\tilde{X},\tilde{X}), (\mathcal{M}_1, \mathcal{M}_2), (\mathcal{M}_3, \mathcal{M}_4)\}$ where $(\mathcal{M}_1, \mathcal{M}_2) = (\langle x, \{x, y\}, \{z\}\rangle, \langle x, \{x, y\}, \emptyset\rangle)$ *and* $(\mathcal{M}_3, \mathcal{M}_4) = (\langle x, \{y\}, \{z\}\rangle, \langle x, \{y\}, \emptyset\rangle)$. $Y = \{y, z\}$, $Y \subseteq X$ $(Y, Y) = (\langle x, \{y\}, \{z\}\rangle, \langle x, \{y\}, \{z\}\rangle) \in \Psi$. Take $C_1 = \{(\tilde{\emptyset},\tilde{\emptyset}), (\mathcal{M}_1, \mathcal{M}_2)\}$, so $(Y, Y) \subseteq (\mathcal{M}_1, \mathcal{M}_2)$ is Double I-open cover from X Double cover Y, so this Double cover satisfy the definition of Double I- compact subspace. Now, we introduce all Double I-open cover for X as follows: $C_2 = \{(\tilde{\emptyset},\tilde{\emptyset}), (\mathcal{M}_3, \mathcal{M}_4)\}$, *so* $(Y,Y) \subseteq (\mathcal{M}_3, \mathcal{M}_4)$; $C_3 = \{(\mathcal{M}_1, \mathcal{M}_2), (\mathcal{M}_3, \mathcal{M}_4)\}$; $C_4 = \{(\tilde{\emptyset},\tilde{\emptyset}), (\tilde{X},\tilde{X})\}$, *so* $(Y,Y) \subseteq (\tilde{X},\tilde{X})$; $C_5 = \{(\tilde{\emptyset},\tilde{\emptyset}), (\tilde{X},\tilde{x}), (\mathcal{M}_1, \mathcal{M}_2)\}$; $C_6 = \{(\tilde{\emptyset},\tilde{\emptyset}), (\tilde{X},\tilde{X}), (\mathcal{M}_3, \mathcal{M}_4)\}$; $C_7 = \{(\tilde{\emptyset},\tilde{\emptyset}), (\tilde{X},\tilde{X}), (\mathcal{M}_1, \mathcal{M}_2), (\mathcal{M}_3, \mathcal{M}_4)\} = \Psi$; $C_8 = \{(\tilde{X},\tilde{X}), (\mathcal{M}_1, \mathcal{M}_2)\}$; $C_9 = \{(\tilde{X},\tilde{X}), (\mathcal{M}_3, \mathcal{M}_4)\}$. In addition, every Double cover of them has a finite sub cover. Hence, Y is Double I-compact sub space of (X , Ψ) .

Theorem 3.19 Let (X, Ψ) be DITS. In addition, $\{(G_{1\alpha}, G_2): \alpha \in \Lambda\}$ be a finite family of Double I- compact sub space of X, then $\cup_{\alpha \in \Lambda}(G_{1\alpha}, G_{2\alpha})$ is Double I-compact.

Proof Let $((Q, D), (C, H)) \subseteq X$ are Double I-compact. Now $(Q, D) \cup (C, H)$ we want to show that this union is Double I-compact. Since $(Q, D) \cup (C, H) \subseteq \cup_{\alpha \in \Lambda}(G_{1\alpha}, G_{2\alpha}) \in \Psi$, $(Q, D) \subseteq \cup_{\alpha \in \Lambda}(G_{1\alpha}, G_{2\alpha})$ *and* $(C, H) \subseteq \cup_{\alpha \in \Lambda}(G_{1\alpha}, G_{2\alpha})$ (since $(Q, D) \subseteq (Q, D) \cup (C, H)$, $(C, H) \subseteq (Q, D) \cup (C, H)$. *Thus*, $\{(G_{1\alpha}, G_2): \alpha \in \Lambda\}$ *is Double cover of* (Q, D) *and* (C, H). *But* (Q, D), (C, H) both Double I-compact sub space of X, then there exist $\alpha_1, \alpha_2, \ldots, \alpha_n$; $(Q, D) \subseteq \cup_{j=1}^{n}(G_{1\alpha j}, G_{2\alpha j})$ and there exist $\alpha_1, \alpha_2, \ldots, \alpha_m$; $(C, H) \subseteq \cup_{h=1}^{m}(G_{1\alpha h}, G_{2\alpha h})$, $(Q, D) \cup (C, H) \subseteq (\cup_{j=1}^{n}(G_{1\alpha j}, G_{2\alpha j})) \cup (\cup_{h=1}^{m}(G_{1\alpha h}, G_{2\alpha h}))$. Therefore, $\{(G_{1\alpha}, G_2): \alpha \in \Lambda\}$ has a finite sub cover for $(Q, D) \cup (C, H)$, $(Q, D) \cup (C, H)$ is Double I-compact.

Theorem 3.20 A Double intuitionistic topological sub space (Y, Ψ_Y) in DITS (X, Ψ) is Double I- compact if and only if Y is Double I-compact in (X,Ψ) .

Proof Assume that (Y, Ψ_Y) is Double I-sub space of (X, Ψ), so that Y is Double I-compact of (Y, Ψ_Y) to prove Y is Double I-compact of (X,Ψ). Suppose that $\{(G_{1\alpha}, G_{2\alpha}): \alpha \in \Lambda\}$ is Double I-open cover of Y where $\{(G_{1\alpha}, G_{2\alpha}): \alpha \in \Lambda\} \in \Psi$ of X which $(Y,Y) \subseteq \cup_{\alpha \in \Lambda}(G_{1\alpha}, G_{2\alpha})$ where $Y = (Y, Y)$ and from that we find $(Y, Y) \subseteq (Y, Y) \cap (\cup_{\alpha \in \Lambda}(G_{1\alpha}, G_{2\alpha})) = \cup_{\alpha \in \Lambda}((Y, Y) \cap (G_{1\alpha}, G_{2\alpha})) = \cup_{\alpha \in \Lambda}(F_{1\alpha}, F_{2\alpha})$ where $(F_{1\alpha}, F_{2\alpha}) = (Y, Y) \cap (G_{1\alpha}, G_{2\alpha})) \in \Psi_Y$. Thus, the collection $\{(F_{1\alpha}, F_{2\alpha}): \alpha \in \Lambda\}$ is Double cover of (Y, Y) with Double I-open set of (Y, Y). Hence there is a finite sub cover $\{(F_{1\alpha j}, {}_{2\alpha j}): j=1, 2\ldots, n\}$ such that $(Y, Y) \subseteq \cup_{j=1}^{n}(F_{1\alpha j}, F_{2\alpha j}) = ((Y, Y) \cap (\cup_{j=1}^{n}(G_{1\alpha j}, G_{2\alpha j})) \subseteq \cup_{j=1}^{n}(G_{1\alpha j}, G_{2\alpha j})$. This means that Y is Double I-compact in (X,Ψ) .

Conversely: Assume that Y is Double I-compact of (X, Ψ), $\{(G_{1\alpha}, G_{2\alpha}): \alpha \in \Lambda\}$ is Double cover of Y with Double I-open set of Y, so $(Y,Y) \subseteq \cup_{\alpha \in \Lambda}(G_{1\alpha}, G_{2\alpha})$; $(G_{1\alpha}, G_{2\alpha}) \in \Psi_Y$, *where* $Y = (Y,Y)$ by definition Double I-subspace for each $\alpha \in \Lambda$ there exist $(F_{1\alpha}, F_{2\alpha}) \in \Psi$ such that $(G_{1\alpha}, G_{2\alpha}) = (Y, Y) \cap (F_{1\alpha}, F_{2\alpha})$, $(Y,Y) \subseteq \cup_{\alpha \in \Lambda}(G_{1\alpha}, G_{2\alpha}) = \cup_{\alpha \in \Lambda}((Y, Y) \cap (F_{1\alpha}, F_{2\alpha})) = (Y, Y) \cap (\cup_{\alpha \in \Lambda}(F_{1\alpha}, F_{2\alpha})) \subseteq \cup_{\alpha \in \Lambda}(F_{1\alpha}, F_{2\alpha})$, $(F_{1\alpha}, F_{2\alpha}) \in \Psi$. Thus the collection $\{(F_{1\alpha}, F_{2\alpha}): \alpha \in \Lambda\}$ is Double cover of (Y,Y) with Double I-open set of X , and where (Y,Y) is Double I-compact of (X,Ψ), so there exist a finite sub cover $\{(F_{1\alpha j}, F_{2\alpha j}): j=1, 2\ldots,n\}$ such that $(Y,Y) \subseteq \cup_{j=1}^{n}(F_{1\alpha j}, F_{2\alpha j})$, $(Y,Y) \subseteq (Y, Y) \cap (\cup_{j=1}^{n}(F_{1\alpha j}, F_{2\alpha j})) = \cup_{j=1}^{n}((Y, Y) \cap (F_{1\alpha j}, F_{2\alpha j})) = \cup_{j=1}^{n}(G_{1\alpha j}, G_{2\alpha j})$ is a finite sub cover of (Y, Y). Therefore, Y is Double I-compact of (Y, Ψ_Y).

Theorem 3.21 Let (X, Ψ) be DITS. Then, (X, Ψ) I-compact if and only if $(X, \Psi \times \Psi)$ is Double I- compact.

Proof Suppose that (X, Ψ) be I-compact, let $\{(G_{1\alpha}, G_{2\alpha}): \alpha \in \Lambda\}$ be a Double I-open cover of X, so $(\tilde{X},\tilde{X}) = \cup_{\alpha \in \Lambda}(G_{1\alpha}, G_{2\alpha})$. Then for each $(G_{1\alpha}, G_{2\alpha})$ *in* $\{(G_{1\alpha}, G_{2\alpha}): \alpha \in \Lambda\}$ there exist $(G_{1\alpha}, G_{2\alpha}) \in \Psi$, $G_{1\alpha} \subseteq G_{2\alpha}$. Thus the collection $\{G_{1\alpha}: \alpha \in \Lambda\} \subseteq \Psi$ is I-open cover of X , so $\tilde{X} = \cup_{\alpha \in \Lambda} G_{1\alpha}$. *Since* (X, Ψ) is I-compact, then there exist a finite sub collection $\{G_{1\alpha j}: j=1, 2\ldots,n\}$ *of* $\{G_{1\alpha}: \alpha \in \Lambda\}$ such that $\tilde{X} = \cup_{j=1}^{n} G_{1\alpha j}$. Now $G_{1\alpha j} \subseteq G_{2\alpha j}$, $\cup_{j=1}^{n} G_{1\alpha j} \subseteq \cup_{j=1}^{n} G_{2\alpha j}$. *Hence* $(\tilde{X},\tilde{X}) = (\cup_{j=1}^{n} G_{1\alpha j}, \cup_{j=1}^{n} G_{2\alpha j}) = \cup_{j=1}^{n}(G_{1\alpha j}, G_{2\alpha j})$. Therefore $(X, \Psi \times \Psi)$ is Double I-compact.

Conversely: Let $(X, \Psi \times \Psi)$ is Double I-compact, let $\{G_{\alpha}: \alpha \in \Lambda\} \subseteq \Psi$ be an I-open cover of X, so the collection $\{(G_{\alpha}, G_{\alpha}): \alpha \in \Lambda\} \subseteq \Psi \times \Psi$ is Double I-open cover of X. Hence, there exist a finite sub collection $\{(G_{\alpha j}, G_{\alpha j}): j=1, 2..., n\}$ *of* $\{(G_{\alpha}, G_{\alpha}): \alpha \in \Lambda\}$ such that $(\tilde{X},\tilde{X}) = \cup_{j=1}^{n} (G_{\alpha j}, G_{\alpha j})$ and so $\tilde{X} = \cup_{j=1}^{n} G_{\alpha j}$. Therefore (X, Ψ) I-compact.

Definition 3.22 Let (X, Ψ) be DITS, let $(Z_1, Z_2) = \{(K_{1\alpha}, K_{2\alpha}): \alpha \in \Lambda\}$ be a family of I-sets, we call this family satisfy the Double intuitionistic finite intersection property (Double I-FIP, for short) if for every finite sub collection $\{(K_{1\alpha j}, K_{2\alpha j}): j=1, 2...,n\}$ *of* $(Z_1. Z_2)$,we have $(K_{1\alpha j}, K_{2\alpha j}) \neq (\tilde{\emptyset},\tilde{\emptyset})$.

Example 3.23 Recall example 3.9. Notes that the intersection every finite number of the family is non-empty, so it satisfies Double I-finite intersection property.

Theorem 3.24 Let (X, Ψ) be DITS. Then, (X, Ψ) Double I-compact if and only if every family of Double I-closed subsets of X satisfy (Double I-FIP) being intersection nonempty.

Proof Assume that (X, Ψ) is Double I-compact space, let $\{(K_{1\alpha}, K_{2\alpha}): \alpha \in \Lambda\}$ be a family of Double I-closed sets satisfy (Double I-FIP) to prove $\cap_{\alpha \in \Lambda} (K_{1\alpha}, K_{2}) \neq (\tilde{\emptyset},\tilde{\emptyset})$. Suppose $\cap_{\alpha \in \Lambda} (K_{1\alpha}, K_{2\alpha}) = (\tilde{\emptyset},\tilde{\emptyset})$, $(\cap_{\alpha \in \Lambda} (K_{1\alpha}, K_{2})^{c} = (\tilde{\emptyset},\tilde{\emptyset})^{c}$, $\cup_{\alpha \in \Lambda} (K_{1\alpha}, K_{2\alpha})^{c} = (\tilde{X},\tilde{X})$. Since $(K_{1\alpha}, K_{2})$ is Double I- closed, so $\forall \alpha$, $(K_{1\alpha}, K_{2\alpha})^{c} \in \Psi$ $\forall \alpha$, $\{(K_{1\alpha}, K_{2\alpha})^{c}: \alpha \in \Lambda\}$ is Double I-open cover of X . Since (X, Ψ) is Double I-compact, thus there exist $\alpha_1, \alpha_2, ...,\alpha_n \in \Lambda\}$, $(\tilde{X},\tilde{X}) = \cup_{j=1}^{n} (K_{1\alpha j}, K_{2\alpha j})^{c}$, $(\tilde{X},\tilde{X})^{c} = (\cup_{j=1}^{n} (K_{1\alpha j}, K_{2\alpha j})^{c})^{c}$, $(\tilde{\emptyset},\tilde{\emptyset}) = \cap_{j=1}^{n} (K_{1\alpha j}, K_{2\alpha j})$ which a contradiction, since this family satisfy (Double I-FIP) , then $\cap_{j=1}^{n} (K_{1\alpha j}, K_{2\alpha j}) \neq (\tilde{\emptyset},\tilde{\emptyset})$. Therefore $\cap_{\alpha \in \Lambda} (K_{1\alpha}, K_{2}) \neq (\tilde{\emptyset},\tilde{\emptyset})$.

Conversely: Assume that $\{(K_{1\alpha}, K_{2\alpha}) : \alpha \in \Lambda\}$ be a family of Double I-closed sets satisfy (Double I-FIP) and $\cap_{\alpha \in \Lambda} (K_{1\alpha}, K_{2\alpha}) \neq (\tilde{\emptyset},\tilde{\emptyset})$ (for any family of Double I-closed sets satisfy (Double I-FIP)). To prove that (X, Ψ) Double I-compact. Suppose that (X, Ψ) is not Double I-compact, so there exist Double I-open cover of X has not finite sub cover, *i.e.*, $(\tilde{X},\tilde{X}) = \cup_{\alpha \in \Lambda} (L_{1\alpha}, L_{2\alpha}) \wedge (\tilde{X},\tilde{X}) \neq \cup_{j=1}^{n} (L_{1\alpha j}, L_{2\alpha j})$; $\forall \alpha_1, \alpha_2, ...,\alpha_n \in \Lambda$, $(\tilde{X},\tilde{X})^{c} \neq (\cup_{j=1}^{n} (L_{1\alpha j}, L_{2\alpha j}))^{c}$, $(\tilde{\emptyset},\tilde{\emptyset}) \neq \cap_{j=1}^{n} (L_{1\alpha j}, L_{2\alpha j})^{c}$ *and* $(L_{1\alpha j}, L_{2\alpha j})^{c} \in \Psi^{c}$, since $(L_{1\alpha j}, L_{2\alpha j}) \in \Psi$ we have the family of Double I-closed sets $\{(L_{1\alpha}, L_{2\alpha})^{c}: \alpha \in \Lambda\}$ satisfy (Double I-FIP), but intersection this family is empty . Since $(\tilde{X},\tilde{X}) = \cup_{\alpha \in \Lambda} (L_{1\alpha}, L_{2\alpha})$, $(\tilde{X},\tilde{X})^{c} = (\cup_{\alpha \in \Lambda} (L_{1\alpha}, L_{2\alpha}))^{c}$, $(\tilde{\emptyset},\tilde{\emptyset}) = \cap_{\alpha \in \Lambda} (K_{1\alpha}, K_{2\alpha})^{c}$ whish a contradiction (with hypothesis) . Therefore (X, Ψ) is Double I-compact space.

Conclusions

In this article, we got the next results: We presented a new set of concepts related to the Double intuitionistic compact space in DITS, and its relationship with some other classes and give examples for each concept. We also proved the family topology space of all Double intuitionistic open sets in X is DITS, every Double I-closed set is Double I-compact, if (X,Ψ) is Double I-compact if and only if (X,T_1) is I-compact (respectively, (X, Ψ_1) is a Ψ_1 Double I-compact where $T_1 = \{Q: (Q, D) \in \Psi\}$ (resp. $\Psi_1 = \{(Q, Q), (Q, D) \in \Psi\}$), *and if* Ψ_2 Double I-compact if and only if (X, T_2) is an T_2 I-compact where $\Psi_2 = \{(D, D), (Q, D)\} \in \Psi$. *Also, if* (X,Ψ) is Double I-compact, then (X,T_3) is I-compact where $T_3 = \{Q :(Q, X) \in \Psi\}$, *if* (X, Ψ) I-compact if and only if $(X, \Psi \times \Psi)$ is Double I- compact. Finally, if (X, Ψ) is Double I-compact if and only if every family of Double I-closed subsets of X satisfy (Double I-FIP) being intersection nonempty.

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الفضاء المرصوص الحدسي المزدوج في الفضاءات التبولوجية الحدسية المزدوجة

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الخلاصة:

الهدف من هذا البحث هو بناء المفهوم الأساسي للتراص في الفضاءات التبولوجية الحدسية المزدوجة (X, Ψ)، حيث قدمنا مفاهيم التراص الحدسي المزدوج وبعض أنواعه. أيضا، اوجدنا العلاقات بين هذه الاصناف مع البراهين والامثلة.