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Solution of Multi-Objective Linear Programming Problem by Advanced Transformation Technique

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Abstract

In this paper, we show a new approach for solving multi-objective linear programming problems (MOLPP). We changed multi-objective linear programming problem into single -objective linear programming problem using advanced transformation technique. This strategy is comparatively simple to calculate. The methodology for the proposed calculation is illustrative with numerical examples, the results appeared that the proposed procedure gives more better than the other strategies and it takes less time to execute.

1. Introduction

In later a long time, the benefits of utilizing optimization strategies to solve decision issues have been broadly recognized in preservation science. For illustration, optimization strategies have been created to best allocate limited resources to secure threatened species. Multi Objective optimization is frequently used when the optimization is with respect to two or more criteria some of which are in conflict with others, e.g. lowering the unit production cost of a product while decreasing the unit production time, etc .

The results of these studies can give to several practices, for example, in order to determine the embodied energy for a product, several energy metrics will be determined within the eco-improvement architecture. in addition to other criteria such as quality and time, energy metrics will be used in the system modelling stage as key performance indicators for the system under consideration. these measures will be used to construct a multi objective optimization model so as to optimize the performance of the system subject to some functional constraints of the current operational environment [1].

Multi-objective Programming (MOP) may be confronted as the expansion of classical single objective programming to the cases in which more than one objective function is unequivocally considered in mathematical optimization models. However, if these functions are conflicting, a paradigm is at stake. The concept of optimal solution not makes sense since, in common, there is no feasible solution that at the same time optimizes all objective functions [2]. Issues with multiple objectives and criteria are by and large knows as multiple criteria optimization or different criteria decision-making (MCDM) problems. These sorts focus on linear programming strategies [3].

A few strategies to eliminate such issues are proposed such as, in (1983) Chandra studied anew approach objective planning [4]. In (2009), Sulaiman & Gulnar found a solution of the multi objective programming problem utilizing mean and median value [5]. In (2016) Sulaiman and Mustafa, using harmonic mean to solve multi-objective linear Programming issues [6]. In (2017) Uday & Shashi, by closet interim guess of fuzzy number and interval programming solving fully fuzzy multi-objective linear programming problem [7]. In (2017) Samsun and Abdul Alim, suggested a new statistical averaging technique for solving MOLPP [8].

In order to expanded this work, we have characterized a multi-objective linear programming issue and examined the calculation to illuminate linear programming issue for multi-objective capacities.

2. Multi-Objective Linear Programming Problem:

The numerical shape of Multi-Objective Linear Programming problem is characterized as takes after:

$$\left.\begin{array}{r} Max.Z_1 = d_1x_1 + \beta_1 \\ Max.Z_2 = d_2x_2 + \beta_2 \\ \vdots \\ Max.Z_r = d_rx_r + \beta_r \\ Min.Z_{r+1} = d_{r+1}x_{r+1} + \beta_{r+1} \\ Min.Z_{r+2} = d_{r+2}x_{r+2} + \beta_{r+2} \\ \vdots \\ Min.Z_s = d_sx_s + \beta_s \end{array}\right\} \quad (2.1)$$

$$\text{s.t: } AX \begin{bmatrix} \geq \\ = \\ \leq \end{bmatrix} B \quad (2.2)$$

$$X \geq 0 \quad (2.3)$$

where x is an n-dimensional vector of choice variables d is n-dimensional vector of constants, B is m-dimensional vector of constants, r is the number of objective function to be maximized, s the number of objective function to maximized plus minimized, $(s-r)$ is the number of objective that is to be minimized, A could be an $(m \times n)$ matrix of coefficients all vectors are expected to be column vectors unless transposed, β_i $(i = 1, 2, \ldots, s)$ are scalar constants.

3. Formulation of Multi-Objective Functions:

The same approach which was taken by Sen. (1983) [4] is followed here to formulate the constraint objective function for the MOLFPP. Assume, we optimize (maximize or minimize) all the objective functions independently in eq.'s (2.1), (2.2) and (2.3) and get the values as follows:

$$\left.\begin{array}{r} Max.Z_1 = \theta_1 \\ Max.Z_2 = \theta_2 \\ . \\ . \\ . \\ Max.Z_r = \theta_r \\ Min.Z_{r+1} = \theta_{r+1} \\ Min.Z_{r+2} = \theta_{r+2} \\ . \\ . \\ . \\ Min.Z_s = \theta_s \end{array}\right\} \quad (3.1)$$

Where, θ_i $(i = 1, 2, \dots, s)$ are the values of objective functions.

Applied Chandra Sen's technique to solve MOLFPP, which is of the form:

$$Max.Z = \sum_{i=1}^{r} \frac{Z_i}{|\theta_i|} - \sum_{i=r+1}^{n} \frac{Z_i}{|\theta_i|} \quad (3.2)$$

And solve eq. (3.2) by Simplex Method.

4. Solving MOLPP using Advanced Transformation Technique:

We can Formulate the combined objective function in eq. (2.1) by our technique to transform (MOLPP) into (SOLPP) as follows:

$$Max.Z = \frac{\sum_{i=1}^{r} Max.Z_i - \sum_{i=r+1}^{s} Min.Z_i}{O_{AT}} \quad (4.1)$$

Subject to the constraints eq. (2.2) and eq. (2.3).

Where, O_{AT} is the Advanced Transformation Technique,

$$O_{AT} = \frac{1}{\frac{1}{m}}, m \neq 0$$

$m = \min\{m_1, m_2\}$, where, $m_1 = \min\{|\theta_i|\}$ for $(i = 1, 2, \dots, r)$ and $m_2 = \min\{|\theta_i|\}$ for $(i = r + 1, r + 2, \dots, s)$.

4.1 Algorithm:

By a few steps we clarify our algorithm;

Step 1: Find the value of each objective function which is to be maximized or minimized.

Step 2: Check the possibility of the solution that gotten in step1, on the off chance that it is doable go to step3, else use dual simplex method to evacuate infeasibility.

Step 3: Allot a name to the optimum value of the objective function Z_i, say θ_i for $i = 1,2,3,\dots,s$

Step 4: Select $m_1 = \min.\{\theta_i\}, \forall i = 1,2,\dots,r., m_2 = \min.\{\theta_i\}, \forall i = r+1, r+2,\dots,s$. And $m = \min\{m_1, m_2\}$, then calculated $O_{AT} = \frac{1}{\frac{1}{m}}$

Step 5: By formula of (4.1) can Optimize the combined objective function beneath the same limitations eq. (2.2) and eq. (2.3).

5. Numerical Examples:

Example 5.1: Consider the MOLP Problem

$Max.Z_1(x) = 5x_1 + 2x_2$

$Max.Z_2(x) = x_1 + x_2$

$Min.Z_3(x) = -2x_1 - 3x_2$

$Min.Z_4(x) = -5x_1 - 4x_2$

Subject to: $x_1 + x_2 \le 5$; $3x_1 + x_2 \le 3$; $x_1, x_2 \ge 0$

Solution 5.1:

First, solve each objective function individually, that is shown in table 1

Table 1: The value of the objective functions and calculate O_{AT}

i	(x_1, x_2)	$Z_i = \lvert\theta_i\rvert$	m_1	m_2	O_{AT}
1	(0,3)	6			
2	(0,3)	3	3		
3	(0,3)	9		9	3
4	(0,3)	12			

Solve **example 5.1** by formula (4.1) we get:

$\sum_{i=1}^{r} Max.Z_i = 6x_1 + 3x_2$, $\sum_{i=r+1}^{s} Min.Z_i = -7x_1 - 7x_2$

$Max.Z = \frac{\sum_{i=1}^{r} Max.Z_i - \sum_{i=r+1}^{s} Min.Z_i}{O_{AT}} = \frac{13x_1 + 10x_2}{3}$, then solve this objective function with the same constraints we get, $Max.Z = 10$ at $(0,3)$.

Solve **example 5.1** by another method such as:

1) By Chandra Sen the result is $Max.Z = 4$ at $(0,3)$.
2) By Mean and Median we get, $Max.Z = 4$ at $(0,3)$.

3) By Harmonic Mean, $Max. Z = 4.29$ at $(0, 3)$.
4) By Geometric Average, $Max. Z = 4.14$ at $(0, 3)$.
5) By New Arithmetic Average, $Max. Z = 5$ at $(0, 3)$.
6) By New Geometric Average, $Max. Z = 5.77$ at $(0, 3)$.
7) By New Harmonic Average, $Max. Z = 6.667$ at $(0, 3)$.

Example 5.2: Consider the MOLP Problem

$Max. Z_1(x) = 4x_1 + x_2$

$Max. Z_2(x) = x_1 + x_2$

$Max. Z_3(x) = 8x_1 + 4x_2$

$Max. Z_4(x) = 10x_1 + 2x_2$

$Min. Z_5(x) = -5x_1 - 3x_2$

$Min. Z_6(x) = -2x_1 - x_2$

Subject to: $x_1 + x_2 \leq 2$; $3x_1 + 2x_2 \leq 6$; $x_1 \geq 1$; $x_2 \leq 5$; $x_1, x_2 \geq 0$

Solution 5.2:

First, solve each objective function individually, that is shown in table 2

Table 2: The value of the objective functions and calculate O_{AT}

i	(x_1, x_2)	$Z_i = \|\theta_i\|$	m_1	m_2	O_{AT}
1	$(2, 0)$	8			
2	$(2, 0), (1,1)$	2	2		2
3	$(2, 0)$	16			
4	$(2, 0)$	20			
5	$(2, 0)$	10			
6	$(2, 0)$	4		4	

Solve **example 5.2** by formula (4.1) we get:

$\sum_{i=1}^{r} Max. Z_i = 23x_1 + 8x_2$, $\sum_{i=r+1}^{s} Min. Z_i = -7x_1 - 4x_2$

$Max. Z = \frac{\sum_{i=1}^{r} Max. Z_i - \sum_{i=r+1}^{s} Min. Z_i}{O_{AT}} = 15x_1 + 6x_2$, then solve this objective function with the same constraints we get, $Max. Z = 30$ at $(2, 0)$.

Solve **example 5.2** by another method such as:

1) By Chandra Sen the result is $Max. Z = 6$ at $(2, 0)$.
2) By Mean and Median we get, $Max. Z = 6$ and $Max. Z = 5.833$ respectively at $(2, 0)$.
3) By Harmonic Mean, $Max. Z = 10.93$ at $(2, 0)$.
4) By Geometric Average, $Max. Z = 7.65$ at$(2, 0)$.
5) By New Arithmetic Average, $Max. Z = 20$ at $(2, 0)$.

6) By New Geometric Average, $Max.Z = 21.22$ at $(2, 0)$.
7) By New Harmonic Average, $Max.Z = 22.5$ at $(2, 0)$.

Example 5.3: Consider the MOLP Problem

$Max.Z_1(x) = x_1 + x_2$

$Max.Z_2(x) = 3x_1 + 2x_2$

$Max.Z_3(x) = x_1 + 5x_2$

$Max.Z_4(x) = x_1$

$Min.Z_5(x) = -x_1 - 3x_2$

$Min.Z_6(x) = -2x_1 - x_2$

$Min.Z_7(x) = -3x_1 - 6x_2$

$Min.Z_8(x) = -4x_1 - 2x_2$

Subject to: $2x_1 + x_2 \leq 4$; $x_1 - x_2 \leq 2$; $9x_1 + 6x_2 \leq 36$; $x_1 - 2x_2 \leq 6$; $x_1 \leq 3$; $x_2 \leq 2$

$x_1, x_2 \geq 0$

Solution 5.3:

First, solve each objective function individually, that is shown in table 3

Table 3: The value of the objective functions and calculate O_{AT}

i	(x_1, x_2)	$Z_i = \lvert\theta_i\rvert$	m_1	m_2	O_{AT}
1	(1, 2)	3			
2	(1, 2)	7			
3	(1, 2)	11			
4	(2, 0)	2	2		2
5	(1, 2)	7			
6	(1, 2), (2, 0)	4		4	
7	(1, 2)	15			
8	(1, 2), (2, 0)	8			

Solve **example 5.3** by formula (4.1) we get:

$\sum_{i=1}^{r} Max.Z_i = 6x_1 + 8x_2$, $\sum_{i=r+1}^{s} Min.Z_i = -10x_1 - 12x_2$

$Max.Z = \frac{\sum_{i=1}^{r} Max.Z_i - \sum_{i=r+1}^{s} Min.Z_i}{O_{AT}} = 8x_1 + 10x_2$, then solve this objective function with the same constraints we get, $Max.Z = 28$ at $(1, 2)$.

Solve **example 5.3** by another method such as:

1) By Chandra Sen the result is $Max.Z = 7.5$ at $(1, 2)$.
2) By Mean and Median we get, $Max.Z = 7.826$ and $Max.Z = 8.933$ respectively $(1, 2)$.
3) By Harmonic Mean, $Max.Z = 10.8375$ at$(1, 2)$.
4) By Geometric Average, $Max.Z = 9.209$ at$(1, 2)$.
5) By New Arithmetic Average, $Max.Z = 18.667$ at $(1, 2)$.
6) By New Geometric Average, $Max.Z = 19.802$ at $(1, 2)$.
7) By New Harmonic Average, $Max.Z = 21$ at $(1, 2)$.

Example 5.4: Consider the MOLP Problem [5]

$Max.z_1 = 3x_1 + 2x_2$

$Max.z_2 = 4x_1 + x_2$

$Max.z_3 = 4x_1 - 2x_2$

$Max.z_4 = 15x_1 + 4x_2$

$Min.z_5 = -6x_1 + 2x_2$

$Min.z_6 = -9x_1 + 3x_2$

$Min.z_7 = -5x_1 + 2x_2$

Subject to: $x_1 + x_2 \le 4;\ x_1 - x_2 \le 2;\ x_1, x_2 \ge 0$

Solution 5.4:

First, solve each objective function individually, that is shown in table 4

Table 4: The value of the objective functions and calculate O_{AT}

i	(x_1, x_2)	$Z_i = \lvert\theta_i\rvert$	m_1	m_2	O_{AT}
1	(3, 1)	11	11		
2	(3, 1)	13			
3	(3, 1)	10			
4	(3, 1)	49			11
5	(3, 1)	-16			
6	(3, 1)	-24			
7	(3, 1)	-13		13	

Solve **example 5.4** by formula (4.1) we get:

$\sum_{i=1}^{r} Max.Z_i = 26x_1 + 5x_2$, $\sum_{i=r+1}^{s} Min.Z_i = -20x_1 + 7x_2$

$Max.Z = \frac{\sum_{i=1}^{r} Max.Z_i - \sum_{i=r+1}^{s} Min.Z_i}{O_{AT}} = \frac{46x_1 - 2x_2}{11}$, then solve this objective function with the same constraints we get, $Max.Z = 12.36$ at $(3, 1)$.

Solve **example 5.4** by another method such as:

1) By Chandra Sen the result is $Max.Z = 7.02$ at $(3, 1)$.
2) By Mean and Median we get, $Max.Z = 6.98$ and $Max.Z = 9.92$ respectively $(3, 1)$.

3) By Harmonic Mean, $Max. Z = 9.16$ at $(3, 1)$.
4) By Geometric Average, $Max. Z = 8.19$ at $(3, 1)$.
5) By New Arithmetic Average, $Max. Z = 11.33$ at $(3, 1)$.
6) By New Geometric Average, $Max. Z = 11.37$ at $(3, 1)$.
7) By New Harmonic Average, $Max. Z = 11.43$ at $(3, 1)$.

Example 5.5: Consider the MOLP Problem [6][8]

$Max. z_1 = x_1 + 2x_2$

$Max. z_2 = x_1$

$Min. z_3 = -2x_1 - 3x_2$

$Min. z_4 = -x_2$

Subject to: $6x_1 + 8x_2 \leq 48;\ x_1 + x_2 \geq 3, x_1 \leq 4, x_2 \leq 3;\ x_1, x_2 \geq 0$

Solution 5.5:

First, solve each objective function individually, that is shown in table 4

Table 5: The value of the objective functions and calculate O_{AT}

i	(x_1, x_2)	$Z_i = \|\theta_i\|$	m_1	m_2	O_{AT}
1	$(4, 3)$	10			
2	$(4, 3)$	4	4		
3	$(4, 3)$	-17			
4	$(4, 3)$	-3		3	3

Solve **example 5.5** by formula (4.1) we get:

$\sum_{i=1}^{r} Max. Z_i = 26x_1 + 5x_2$, $\sum_{i=r+1}^{s} Min. Z_i = -20x_1 + 7x_2$

$Max. Z = \frac{\sum_{i=1}^{r} Max. Z_i - \sum_{i=r+1}^{s} Min. Z_i}{O_{AT}} = \frac{46x_1 - 2x_2}{11}$, then solve this objective function with the same constraints we get, $Max. Z = 11.33$ at $(3, 1)$.

Solve **example 5.5** by another method such as:

1) By Chandra Sen the result is $Max. Z = 3.9998$ at $(4, 3)$.
2) By Mean and Median we get, $Max. Z = 4$ and $Max. Z = 4$ respectively $(4, 3)$.
3) By Harmonic Mean, $Max. Z = 6.37103$ at $(4, 3)$.
4) By Geometric Average, $Max. Z = 5.0141$ at $(4, 3)$.
5) By New Arithmetic Average, $Max. Z = 9.7141$ at $(4, 3)$.
6) By New Geometric Average, $Max. Z = 9.81415$ at $(4, 3)$.
7) By New Harmonic Average, $Max. Z = 9.8593$ at $(4, 3)$.

6. Comparison:

Table 6 summarizes the results of the MOLPP by using some techniques. It shows the comparison between the technique that studied in this paper and other techniques. The solution of the MOLPP that obtained by our technique (Advanced Transformed Technique) is better than the other techniques that studied previously as shown in Table 6.

Table 6: Comparison between results

Techniques	Ex.5.1	Ex.5.2	Ex.5.3	Ex.5.4	Ex. 5.5
Chandra Sen	4	6	7.5	7.02	3.9998
Mean	4	6	7.826	6.98	4
Median	4	5.833	8.933	9.92	4
Harmonic average	4.29	10.93	10.8375	9.16	6.37103
Geometric average	4.14	7.65	9.209	8.19	5.0141
New Arithmetic Average	5	20	18.667	11.33	9.7141
New Geometric average	5.77	21.22	19.802	11.37	9.81415
New Harmonic Average	6.667	22.5	21	11.43	9.8593
Advanced Transformed Technique	10	30	28	12.36	11.33

7. Conclusion

In this paper, we displayed a method to convert MOLPP into SOLPP, we use Advanced Transformed Technique and some technique that studied previously for solving MOLPP. Then compare the results that obtained by our technique and other technique, the comparison of these methods is based on the value of the objective function. In example 5.1 when using Advanced Transformed Technique, we have obtained (Z=10) which are better than the other techniques (Chandra Sen, Mean & Median, Harmonic average, Geometric average, New Arithmetic Average, New Geometric average and New Harmonic Average) which is (4, 4, 4.29, 4.14, 5, 5.77 and 6.667) respectively. Thus, after solving the numerical examples, are presented in Table 6, found that the solution of MOLPP by Advanced Transformed Technique is better than the other techniques.

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حل مشكلة البرمجة الخطية متعددة الأهداف بتقنية التحويل المتقدمة

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الخلاصة:

في هذا البحث، نقدم طريقة جديدة لحل مشاكل البرمجة الخطية متعددة الأهداف . (MOLPP)لقد قمنا بتحويل مشكلة البرمجة الخطية متعددة الأهداف إلى مشكلة البرمجة الخطية ذات الهدف الواحد باستخدام تقنية التحويل المتقدمة. هذه الاستراتيجية سهلة الحساب نسبيًا. ولقد تم توضيح منهجية الحساب المقترحة بأمثلة عددية، وأظهرت النتائج أن الاستراتيجية المقترحة تعطي طريقة أفضل مقارنة بالطرق الأخرى وأنها تستغرق وقتًا أقل للتنفيذ.