

Approximate Solution of Time- Fractional Hirota-Satsuma Coupled KdV Equations by Hybrid ZZ Transform with Homotopy Perturbation Method

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Abstract

The elementary idea in this effort is to spread the ZZ transform method to resolution the nonlinear fractional partial differential equations by joining them with the homotopy perturbation method (HPM). The new procedure was used to solve Hirota-Satsuma coupled KdV equations approximately, the result was accuracy with at smallest iterations. The numerical results and the graphical representations tell that the projected method does very well in terms of competence and ease. Therefore, it can be applied to solve more problems in the field of non-linear fractional differential equations. The fractional derivative is defined in Caputo sense. To display the strength of the projected method, we present a numerical application, calculate two types of errors, and plan figures of the found results. All implications using MAPLE, 2019.

Introduction

Fractional differential equations are a global procedure of ordinary and partial differential equations [1]. Lately, many studies in sciences and engineering have established that the dynamics of numerous systems in nature can be defined additional exactly by nonlinear fractional partial differential equations (FPDEs), for example, in physics, engineering, biology, etc. [2-5]. The specialists in this field have made great efforts to facilitate dealing with FPDEs by generating some ideas of fractional derivatives, such as Caputo's definition [6].

Maximum of fractional differential equations do not consume exact analytical solutions; so, numerous approaches have been well-known to obtain approximate and analytic solutions of (FPDEs), counting the reproducing kernel method [7], ARA residual power series method [8], the double ARA-Sumudu transform [9], Adomian decomposition method [10], the power series method [11], and others [12-16]

The adapted method is named ZZ transform with Homotopy perturbation method (ZZ-HPM) is a procedure for solving linear and nonlinear FPDEs [17].

The Hirota-Satsuma coupled KdV equations of time-fractional order (HS-KdV) [18]:

$$\begin{aligned} D_t^\alpha u &= \frac{1}{2}u_{xxx} - 3uu_x + 3(vw)_x \\ D_t^\alpha v &= -v_{xxx} + 3uv_x, \quad t > 0, 0 < \alpha \leq 1, \\ D_t^\alpha w &= -w_{xxx} + 3uw_x. \end{aligned} \quad (1)$$

Subject to the initial conditions:

$$\begin{aligned} u(x, 0) &= \frac{\beta - 2k^2}{3} + 2k^2 \tanh^2(kx) \\ v(x, 0) &= -\frac{4k^2 c_0 (\beta + k^2)}{3c_1^2} + \frac{4k^2 (\beta + k^2) \tanh(kx)}{3c_1} \\ w(x, 0) &= c_0 + c_1 \tanh(kx) \end{aligned} \quad (2)$$

where $k, c_0, c_1 \neq 0$, and β are arbitrary constant. The circumstance of $\alpha=1$ in Eq.(1) was answered through Wu et al [19].

Basic Definitions

In this section, there are three basic definitions, and some properties of ZZ transform

Definition .1[20]: Caputo's fractional time derivative is defined as

$$({}^c D_t^\alpha u)(x, t) = \begin{cases} \frac{1}{\Gamma(q-\alpha)} \int_0^t (t-K)^{q-\alpha-1} \frac{\partial^q u(x, K)}{\partial K^q} dK, & q-1 < \alpha < q, \\ \frac{\partial^q u(x, t)}{\partial t^q}, & \alpha = q, \end{cases}$$

where $\alpha \in \mathbb{R}^+, q \in \mathbb{N}^*$.

Definition .2 [17]: Let $g(t)$ be a function defined for all $t \geq 0$. The ZZ transform of $g(t)$ is the function $T(v, s)$ such that

$$Z[g(t)] = T(v, s) = s \int_0^\infty g(vt) e^{-st} dt.$$

Some properties of the ZZ transform

1. $Z[1] = 1$
2. $Z[t] = \frac{v}{s}$
3. $Z[t^n] = n! \frac{v^n}{s^n}, n = 0, 1, 2, \dots$
4. $Z[t^\alpha] = \Gamma(\alpha + 1) \frac{v^\alpha}{s^\alpha}, \alpha \geq 0$
5. $Z[({}^c D_{0+}^\alpha u)(t)] = \frac{s^\alpha}{v^\alpha} Z[u(t)] - \sum_{r=0}^{n-1} \frac{s^{\alpha-r}}{v^{\alpha-r}} u^{(r)}(0), n-1 < \alpha \leq n, n = 1, 2, \dots$

Definition 3. [21]: Least Square Weighted Function (LSWF)

Let the exact solution $y(x, t)$ of a nonlinear FPDEs defined on a closed square $K = [c, h] \times [0, V]$ as follows:

$$Ny(x, t) = g(x, t), \quad (3)$$

where N be a nonlinear differential operator, $c, h \in \mathbb{R}$ and $V > 0$. Assume $Y(x, t)$ is an approximate solution of y . The following residual error $R(x, t)$ is estimated by replaced Y in Eq.(3)

$$R(x, t) = NY(x, t) - g(x, t). \quad (4)$$

The main objective of this definition is to minimalize $R(x, t)$. The least squares weighted method is careful an appropriate numerical measured to determine the amount of error resulting from approximation and is defined as follows:

$$\text{LSWF} = \int_c^h \int_0^V |R(x, t)|^2 dt dx. \quad (5)$$

Basic idea of Hybrid ZZ Transform with Homotopy perturbation Method (ZZ- HPM)

Take ZZ transform for Eq.(1)

$$\begin{aligned} Z[D_t^\alpha u] &= Z\left[\frac{1}{2}u_{xxx}\right] - Z[3uu_x] + Z[3(vw)_x] \\ Z[D_t^\alpha v] &= -Z[v_{xxx}] + Z[3uv_x], \\ Z[D_t^\alpha w] &= -Z[w_{xxx}] + Z[3uw_x]. \end{aligned} \quad (6)$$

By properties of ZZ transform:

$$\begin{aligned} \frac{s^\alpha}{v^\alpha} Z[u(x, t)] - \sum_{k=0}^{n-1} \frac{s^{\alpha-k}}{v^{\alpha-k}} u^k(x, 0) &= Z\left[\frac{1}{2}u_{xxx}\right] - Z[3uu_x] + Z[(3vw)_x], \\ \frac{s^\alpha}{v^\alpha} Z[v(x, t)] - \sum_{k=0}^{n-1} \frac{s^{\alpha-k}}{v^{\alpha-k}} v^k(x, 0) &= -Z[v_{xxx}] + Z[3uv_x], \\ \frac{s^\alpha}{v^\alpha} Z[w(x, t)] - \sum_{k=0}^{n-1} \frac{s^{\alpha-k}}{v^{\alpha-k}} w^k(x, 0) &= -Z[w_{xxx}] + Z[3uw_x]. \end{aligned} \quad (7)$$

So,

$$\begin{aligned} Z[u(x, t)] &= \frac{v^\alpha}{s^\alpha} \sum_{k=0}^{n-1} \frac{s^{\alpha-k}}{v^{\alpha-k}} u^k(x, 0) + \frac{v^\alpha}{s^\alpha} \left\{ Z\left[\frac{1}{2}u_{xxx}\right] - Z[3uu_x] + Z[(3vw)_x] \right\}, \\ Z[v(x, t)] &= \frac{v^\alpha}{s^\alpha} \sum_{k=0}^{n-1} \frac{s^{\alpha-k}}{v^{\alpha-k}} v^k(x, 0) + \frac{v^\alpha}{s^\alpha} \{-Z[v_{xxx}] + Z[3uv_x]\}, \\ Z[w(x, t)] &= \frac{v^\alpha}{s^\alpha} \sum_{k=0}^{n-1} \frac{s^{\alpha-k}}{v^{\alpha-k}} w^k(x, 0) + \frac{v^\alpha}{s^\alpha} \{-Z[w_{xxx}] + Z[3uw_x]\}. \end{aligned} \quad (8)$$

Hence,

$$\begin{aligned} Z[u(x, t)] &= u(x, 0) + \frac{v^\alpha}{s^\alpha} \left\{ Z\left[\frac{1}{2}u_{xxx}\right] - Z[3uu_x] + Z[(3vw)_x] \right\}, \\ Z[v(x, t)] &= v(x, 0) + \frac{v^\alpha}{s^\alpha} \{-Z[v_{xxx}] + Z[3uv_x]\}, \\ Z[w(x, t)] &= w(x, 0) + \frac{v^\alpha}{s^\alpha} \{-Z[w_{xxx}] + Z[3uw_x]\}. \end{aligned} \quad (9)$$

Taking inverse ZZ transform:

$$\begin{aligned} u(x, t) &= u(x, 0) + Z^{-1} \left[\frac{v^\alpha}{s^\alpha} \left\{ Z\left[\frac{1}{2}u_{xxx}\right] - Z[3uu_x] + Z[(3vw)_x] \right\} \right], \\ v(x, t) &= v(x, 0) + Z^{-1} \left[\frac{v^\alpha}{s^\alpha} \{-Z[v_{xxx}] + Z[3uv_x]\} \right], \\ w(x, t) &= w(x, 0) + Z^{-1} \left[\frac{v^\alpha}{s^\alpha} \{-Z[w_{xxx}] + Z[3uw_x]\} \right]. \end{aligned} \quad (10)$$

For applying the homotopy perturbation method in Eq.(10),

$$u(x, t) = \sum_{n=0}^{\infty} P^n u_n(x, t), \quad v(x, t) = \sum_{n=0}^{\infty} P^n v_n(x, t) \quad \text{and} \quad w(x, t) = \sum_{n=0}^{\infty} P^n w_n(x, t), \quad (11)$$

where the homotopy P , is considered as a small parameter $P \in [0,1]$. The nonlinear terms can be decayed as:

$$\begin{aligned} \sum_{n=0}^{\infty} P^n u_n(x, t) &= u(x, 0) + P \left(Z^{-1} \left[\frac{v^\alpha}{s^\alpha} \left\{ Z\left[\frac{1}{2}(\sum_{n=0}^{\infty} P^n u_n)_{xxx}\right] - 3Z[\sum_{n=0}^{\infty} P^n R_n(u)] + \right. \right. \right. \\ &\quad \left. \left. \left. 3Z[\sum_{n=0}^{\infty} P^n B_n(v, w)] + 3Z[\sum_{n=0}^{\infty} P^n T_n(v, w)] \right\} \right] \right), \\ \sum_{n=0}^{\infty} P^n v_n(x, t) &= v(x, 0) + P \left(Z^{-1} \left[\frac{v^\alpha}{s^\alpha} \{-Z[(\sum_{n=0}^{\infty} P^n v_n)_{xxx}] + 3Z[\sum_{n=0}^{\infty} P^n D_n(u, v)]\} \right] \right), \\ \sum_{n=0}^{\infty} P^n w_n(x, t) &= w(x, 0) + P \left(Z^{-1} \left[\frac{v^\alpha}{s^\alpha} \{-Z[(\sum_{n=0}^{\infty} P^n w_n)_{xxx}] + 3Z[\sum_{n=0}^{\infty} P^n E_n(u, w)]\} \right] \right). \end{aligned}$$

Comparing the coefficient of the power of P on both sides in Eq.(12),

$$\begin{aligned}
 P^0: u(x, 0) &= \frac{\beta - 2k^2}{3} + 2k^2 \tanh^2(kx) \\
 P^0: v(x, 0) &= -\frac{4k^2 c_0 (\beta + k^2)}{3c_1^2} + \frac{4k^2 (\beta + k^2) \tanh(kx)}{3c_1} \\
 P^0: w(x, 0) &= c_0 + c_1 \tan h(kx) \\
 &\cdot \\
 &\cdot \\
 &\cdot \\
 P^n: u_n(x, t) &= Z^{-1} \left[\frac{v^\alpha}{S^\alpha} \left\{ Z \left[\frac{1}{2} (u_{n-1})_{xxx} \right] - 3Z[R_{n-1}(u)] + 3Z[B_{n-1}(v, w)] + 3Z[T_{n-1}(v, w)] \right\} \right] \\
 P^n: v_n(x, t) &= Z^{-1} \left[\frac{v^\alpha}{S^\alpha} \left\{ -Z[(v_{n-1})_{xxx}] + 3Z[D_{n-1}(u, v)] \right\} \right] \\
 P^n: w_n(x, t) &= Z^{-1} \left[\frac{v^\alpha}{S^\alpha} \left\{ -Z[(w_{n-1})_{xxx}] + 3Z[E_{n-1}(u, w)] \right\} \right]
 \end{aligned} \tag{13}$$

where, $n \in \mathbb{N}^*$. We stay in this way to find the general recursive relations.

The primary limited components of He's polynomials, are assumed in

$$\begin{aligned}
 R_0(u) &= u_0 u_{0x} \\
 R_1(u) &= u_1 u_{0x} + u_0 u_{1x} \\
 R_2(u) &= u_2 u_{0x} + u_1 u_{1x} + u_0 u_{2x} \\
 B_0(v, w) &= v_0 w_{0x} \\
 B_1(v, w) &= v_1 w_{0x} + v_0 w_{1x} \\
 B_2(v, w) &= v_2 w_{0x} + v_1 w_{1x} + v_0 w_{2x} \\
 T_0(v, w) &= w_0 v_{0x} \\
 T_1(v, w) &= w_1 v_{0x} + w_0 v_{1x} \\
 T_2(v, w) &= w_2 v_{0x} + w_1 v_{1x} + w_0 v_{2x} \\
 D_0(u, v) &= u_0 v_{0x} \\
 D_1(u, v) &= u_1 v_{0x} + u_0 v_{1x} \\
 D_2(u, v) &= u_2 v_{0x} + u_1 v_{1x} + u_0 v_{2x} \\
 E_0(u, w) &= u_0 w_{0x} \\
 E_1(u, w) &= u_1 w_{0x} + u_0 w_{1x} \\
 E_2(u, w) &= u_2 w_{0x} + u_1 w_{1x} + u_0 w_{2x}
 \end{aligned} \tag{14}$$

Lastly, the approximate solution is planned through

$$\begin{aligned}
 u(x, t) &= \lim_{A \rightarrow \infty} \sum_{n=0}^A u_n(x, t), \\
 v(x, t) &= \lim_{A \rightarrow \infty} \sum_{n=0}^A v_n(x, t), \\
 w(x, t) &= \lim_{A \rightarrow \infty} \sum_{n=0}^A w_n(x, t).
 \end{aligned} \tag{15}$$

The convergence of Eq.(15) has been verified in [22, 23]

Numerical Results and discussion

This paper presents a new approximate solution for the Hirota-Satsuma coupled KdV equations. We applied ZZ-HPM to obtain a solution to the target equations. This method has shown high efficiency by comparing our results with exact results in the case of integers. We also analysed the results and drew solutions. In the Tables 1-3, it was found the absolute error for each of the approximate solutions in the case of $\alpha = 1$, where the result of this error was very close to zero and was large and clear. As for Table 4, the error LSWF was found in cases

of different types of α , and its result was also close to zero and clearly, which indicates the efficiency of this method. Finally, in the three-dimensional figures 1-3, the approximate solutions u, v and w were drawn in succession with the exact solution in the case of $\alpha = 1$. It was noted that the degree of closeness between the shapes representing the exact solution and their counterparts representing the proposed approximate solution was observed.

Table 1. The ZZ-HPM solutions when $\alpha = 1, k = 0.1, c_0 = 1.5, c_1 = 0.1, \beta = 1.5, t = 0.2$ for $u(x, t)$, and the exact solutions of u and the absolute error between u_e and u_a .

x	u_{ZZ-HPM}	Exact u	Error $ u_e - u_a $
0	0.4933513225	0.4933513333	1.079449×10^{-8}
0.2	0.4933832501	0.4933832894	3.950084×10^{-8}
0.4	0.4934310141	0.4934310821	6.794698×10^{-8}
0.6	0.4934944625	0.4934945587	9.59281×10^{-8}
0.8	0.4935733945	0.4935735173	1.232621×10^{-7}
1	0.4936675613	0.4936677112	1.497754×10^{-7}

Table 2. The estimated solutions when $\alpha = 1, k = 0.1, c_0 = 1.5, c_1 = 0.1, \beta = 1.5, t = 0.2$ for $v(x, t)$, and the exact solutions of v and the absolute error between v_e and v_a .

x	v_{ZZ-HPM}	Exact v	Error $ v_e - v_a $
0	-3.013961811	-3.013960000	1.811348×10^{-6}
0.2	-3.009941714	-3.009939906	$1.80627614 \times 10^{-6}$
0.4	-3.005929641	-3.005927848	$1.79545942 \times 10^{-6}$
0.6	-3.001928766	-3.001926986	$1.77895530 \times 10^{-6}$
0.8	-2.997942228	-2.997940472	$1.75687424 \times 10^{-6}$
1	-2.993973120	-2.993971391	$1.72937559 \times 10^{-6}$

Table 3. The estimated solutions when $\alpha = 1, k = 0.1, c_0 = 1.5, c_1 = 0.1, \beta = 1.5, t = 0.2$ for $w(x, t)$, and the exact solutions of w and the absolute error between w_e and w_a .

X	w_{ZZ-HPM}	Exact w	Error $ w_e - w_a $
0	1.502999100	1.50300	8.99676×10^{-7}
0.2	1.504995837	1.504996734	$8.97159676 \times 10^{-7}$
0.4	1.506988589	1.506989480	$8.91784584 \times 10^{-7}$
0.6	1.508975778	1.508976661	$8.83586885 \times 10^{-7}$
0.8	1.510955847	1.510956718	$8.72623243 \times 10^{-7}$
1	1.512927258	1.512928116	$8.58963038 \times 10^{-7}$

Table 4. Provided the LSWF quantitive when $t \in [0,1], x \in [-40,40], c_0 = 1.5, c_1 = 0.1, \beta = 1.5$.

Quantities	LSWF		
	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$
E1	$6.25962538 \times 10^{-6}$	$7.01110594 \times 10^{-6}$	$6.49451683 \times 10^{-6}$
E2	$2.09469033 \times 10^{-6}$	$7.76131470 \times 10^{-5}$	$7.44520453 \times 10^{-5}$
E3	$5.16759482 \times 10^{-6}$	$1.91471406 \times 10^{-5}$	$1.83672977 \times 10^{-5}$

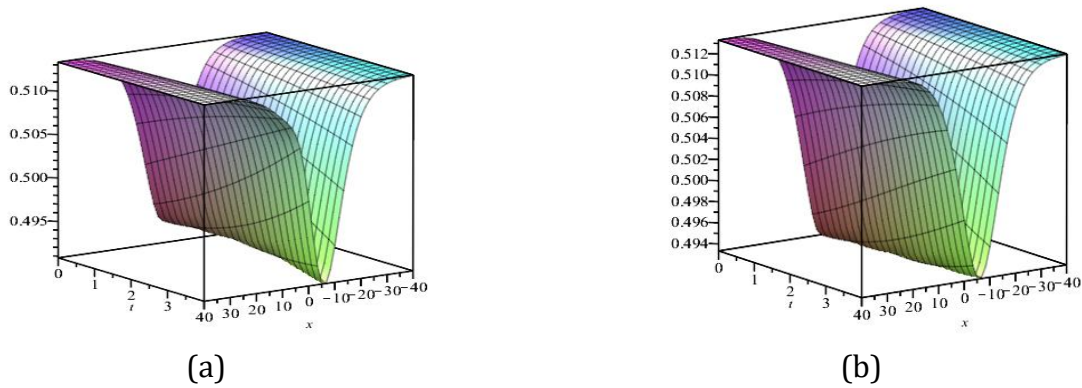


Figure 1. The solution of $u(x, t)$, ZZ-HPM solution in (a) and exact solution in (b), while $k = 0.1$, $\alpha = 1$, $\beta = 1.5$, $c_0 = 1.5$, $c_1 = 0.1$, $t = 0.4$, $x = -40..40$.

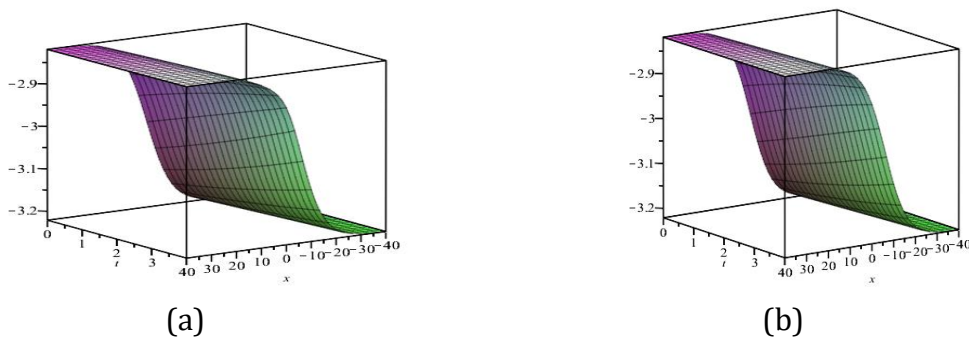


Figure 2. The solution of $v(x, t)$, ZZ-HPM solution in (a) and exact solution in (b), while $k = 0.1$, $\alpha = 1$, $\beta = 1.5$, $c_0 = 1.5$, $c_1 = 0.1$, $t = 0.4$, $x = -40..40$.

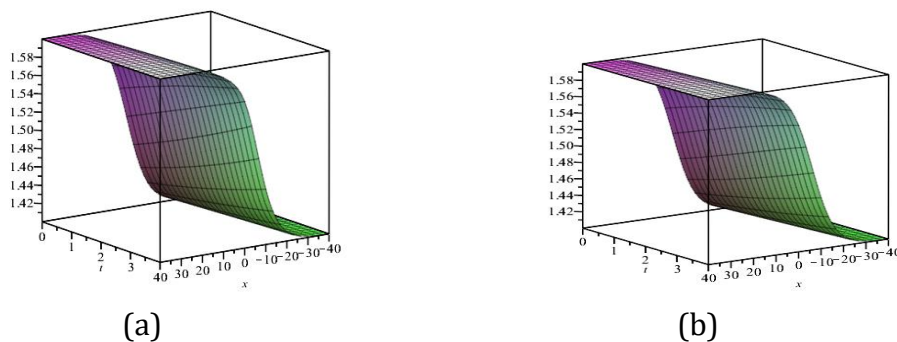


Figure 3. The solution of $w(x, t)$, ZZ-HPM solution in (a) and exact solution in (b), while $k = 0.1$, $\alpha = 1$, $\beta = 1.5$, $c_0 = 1.5$, $c_1 = 0.1$, $t = 0.4$, $x = -40..40$.

Conclusions

The ZZ- HPM with Caputo fractional derivative, was proved very effective in solving time-fractional PDEs. Approximate solutions are obtained with acceptable error and minimum number of iterations. As a result, the conclusion that comes through this work is that ZZ-HPM can be applied to other fractional PDEs of higher-order due to the efficiency and flexibility in the application. Through our work, we accomplish the following:

- ZZ-HPM is an important technique for solving systems of (FPDEs).
- ZZ-HPM is a simple method compared to other numerical methods as it requires fewer calculations.

For upcoming evaluations, the outcomes of this study can be useful to many disciplines. Such as electronic circuits and communication systems.

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الحل التقريبي لمعادلات KdV المقترنة بكسر زمني هيروتا-ساتسوما باستخدام تحويل ZZ الهجين مع اضطراب التماثل

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الخلاصة:

الفكرة الأساسية في هذا العمل هي نشر طريقة تحويل ZZ لحل المعادلات التفاضلية الجزئية الكسرية غير الخطية من خلال ربطها بطريقة اضطراب التماثل (HPM). تم استخدام الإجراء الجديد لحل نظام اقتران هيروتا-ساتسوما (HS-KdV) تقريباً، وكانت النتيجة دقيقة بأقل تكرار. تخبرنا النتائج العددية والتمثيلات الرسومية أن الطريقة المقدمة تعمل بشكل جيد للغاية من حيث الكفاءة والسهولة. لذلك، يمكن تطبيقها لحل المزيد من المسائل في مجال المعادلات التفاضلية الجزئية غير الخطية. يتم تعريف المشتق الكسري حسب مفهوم كابوتو. لإظهار قوة الطريقة المقدمة، نقدم تطبيقاً عددياً، ونحسب نوعين من الأخطاء، ونقدم أشكال النتائج التي تم العثور عليها. تم استخدام MAPLE, 2019 لجميع التطبيقات.

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